

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



## THESIS

**PERFORMANCE ANALYSIS OF BINARY  
FSK SIGNALS WITH L-FOLD DIVERSITY  
SELECTION COMBINING TECHNIQUES IN  
A NAKAGAMI-M FADING CHANNEL**

by

Theofanis Polychronos

September 1998

Thesis Advisor:

Tri T. Ha

Co-Advisor:

Ralph D. Hippenstiel

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19980914 027

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE September 1998		3. REPORT TYPE AND DATES COVERED Master's Thesis
4. TITLE AND SUBTITLE PERFORMANCE ANALYSIS OF BINARY FSK SIGNALS WITH L-FOLD DIVERSITY SELECTION COMBINING TECHNIQUES IN A NAKAGAMI-M FADING CHANNEL			5. FUNDING NUMBERS	
6. AUTHOR(S) Polychronos, Theofanis				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Research and Development Office			10. SPONSORING / MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (maximum 200 words) This thesis investigates the performance analysis of a non-coherent Binary Frequency Shift Keying (BFSK) receiver using Selection Combining techniques over a frequency non-selective, slowly fading Nakagami channel. These techniques are independent of the number of diversity branches, so simpler receivers can be employed. First order selection Combining (SC), second order Selection Combining (SC-2) and third order Selection Combining (SC-3) techniques are evaluated and compared to each other. Numerical results show that the performance improves as the order of Selection Combining techniques increases.				
14. SUBJECT TERMS Nakagami -m fading channel, Diversity Combining Techniques, Selection Combining(SC).			15. NUMBER OF PAGES 74	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	



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IN A NAKAGAMI-M FADING CHANNEL**

Theofanis Polychronos  
Lieutenant, Hellenic Navy  
B.S., Hellenic Naval Academy, 1990

Submitted in partial fulfillment of the  
requirements for the degree of

**MASTER OF SCIENCE IN ELECTRICAL ENGINEERING**

from the

**NAVAL POSTGRADUATE SCHOOL  
September 1998**

Author: T. Polychronos  
Theofanis Polychronos

Approved by: Tri T. Ha  
Tri T. Ha, Thesis Advisor

Ralph D. Hippenstiel  
Ralph D. Hippenstiel, Co-Advisor

Jeffrey B. Knorr  
Jeffrey B. Knorr, Chairman,  
Department of Electrical and  
Computer Engineering



## **ABSTRACT**

This thesis investigates the performance analysis of a non-coherent Binary Frequency Shift Keying (BFSK) receiver using Selection Combining techniques over a frequency non-selective, slowly fading Nakagami channel. These techniques are independent of the number of diversity branches, so simpler receivers can be employed.

First order selection Combining (SC), second order Selection Combining (SC-2) and third order Selection Combining (SC-3) techniques are evaluated and compared to each other. Numerical results show that the performance improves as the order of Selection Combining techniques increases.



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## **ACKNOWLEDGMENTS**

This thesis is dedicated to my wife Nelly, who supports and encourages me in every step of my life, to my parents, who gave me the means to acquire my knowledge, and to my son Timos, who brought joy into my life.



# I. INTRODUCTION

## A. BACKGROUND

Diversity is a well known technique that may be used to reduce the effects of channel fading. Fading is produced when there is interference between two or more versions of the transmitted signal arriving at the receiver at different times [2]. When a diversity technique is used, we transmit or receive several replicas with the same information. In this way we reduce the probability that all replicas of the signal will fade simultaneously. So the demodulation of the information at the receiver can be done using the more reliable replica of the signal. We can employ various diversity techniques. The most well known ones are frequency, time and space diversity.

In frequency diversity the information is transmitted on  $L$  different carrier frequencies. Here the separation between the carriers must equal or exceed the coherence bandwidth  $(\Delta f)_c$  of the channel. The coherence bandwidth  $(\Delta f)_c$  is defined as the range of frequencies over which two frequency components have a strong amplitude correlation [2].

In time diversity the information is transmitted  $L$  times. Here the separation between the successive time slots must equal or exceed the coherence time  $(\Delta t)_c$  of the channel. The coherence time  $(\Delta t)_c$  is defined as the time duration over which two received signals have a strong amplitude correlation [2].

In space diversity a single transmitting antenna and several receiving antennas are employed. These receiving antennas must be placed at least 10 wavelengths apart from each other, so that the multipath components in the signal have different propagation delays at the

antennas.

There are also some other types of diversity such as angle-of-arrival and polarization diversity. But generally frequency diversity, time diversity and space diversity are employed.

At the receiver we use some diversity combining techniques like Equal Gain Combining (EGC) and First, Second and Third Order Selection Combining (SC-1, SC-2, and SC-3 respectively). The different orders of Selection Combining techniques are investigated in this thesis for noncoherent Binary Frequency Shift Keying (BFSK) signals in a slowly fading Nakagami-m channel.

## B. NAKAGAMI FADING CHANNEL

In this thesis, the received signal's amplitude is assumed to be a Nakagami-  $m$  random variable and its probability density function is given by [1]

$$f_a(a) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m a^{2m-1} \exp\left(-\frac{m a^2}{\Omega}\right) \quad , \quad (1)$$

where  $a \geq 0$  and

$$m = \frac{\Omega^2}{\text{var}\{a^2\}} \geq \frac{1}{2} \quad , \quad (2)$$

and where

$$\Omega = E\{a^2\} \quad . \quad (3)$$

The function  $\Gamma(m)$  is defined as

$$\Gamma(m) = \int_0^{\infty} t^{m-1} \exp(-t) dt \quad (4)$$

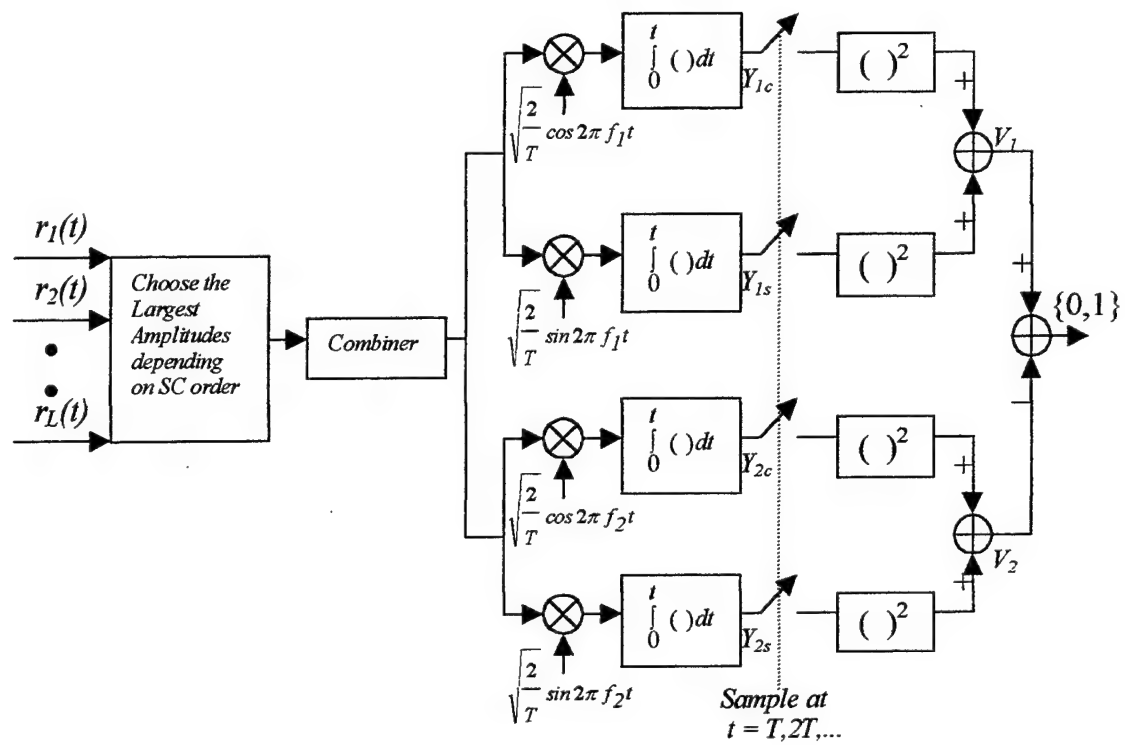
By changing the  $m$ -parameter of the Nakagami distribution we can model different environments. For example, for  $m=1$  we have the Rayleigh fading channel and for  $m=0.5$  we have the one-sided Gaussian fading distribution. Finally, as  $m$  tends to infinity the channel becomes non-fading.

### C. NON-COHERENT BFSK RECEIVER FOR SC TECHNIQUES

Fig. 1 shows the block diagram of the non-coherent BFSK receiver used for the Selection Combining techniques [1]. Depending on the order of Selection Combining that is employed, we choose the signals with the largest amplitudes. For example, if we want to employ Third Order Selection Combining, we choose components with the first three largest amplitudes. Then we combine these signals into one signal, as is described in the next chapters and pass the resulting signal to the demodulator. For the binary orthogonal modulation scheme, as we employ it, non-coherent detection should be performed. Thus we use a square-law detector to demodulate the signal [1].

In BFSK when data bit  $b_i=1$  is transmitted the waveform  $v^{(1)}(t)$  is given by the expression

$$v^{(1)}(t) = \begin{cases} A\sqrt{\frac{2}{T}}\cos(2\pi f_1 t + \theta_1), & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases}, \quad (5)$$



**Figure 1.** Non-coherent BFSK receiver for SC techniques

where  $A\sqrt{2/T}$  is the amplitude of the signal,  $T$  is the bit duration,  $f_1$  is the carrier frequency and  $\theta_1$  is the signal phase.

When data bit  $b_i=0$  is transmitted the waveform  $v^{(2)}(t)$  is given by the expression

$$v^{(2)}(t) = \begin{cases} A\sqrt{\frac{2}{T}}\cos(2\pi f_2 t + \theta_2), & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases}, \quad (6)$$

where again  $A\sqrt{2/T}$  is the amplitude of the signal,  $T$  is the bit duration,  $\theta_2$  is the signal phase and  $f_2$  is the carrier frequency. The two frequencies must have such values that the two waveforms are orthogonal over the interval  $[0, T]$  hence the minimum frequency spacing must be

$$|f_2 - f_1| = \frac{1}{T} \quad (7)$$

The actual received signal in the above time interval is

$$r^{(i)}(t) = aV^{(i)}(t) + n(t), \quad i = 1, 2, \quad (8)$$

where  $a$  is the Nakagami-m random variable with probability density function given by (1) and  $n(t)$  is white Gaussian noise with power spectral density

$$\sigma_n^2 = \frac{N_0}{2} \quad (9)$$



Assuming now that data bit  $b_i=1$  is transmitted, the in-phase outputs of the receiver are

$$Y_{1c} = aA \cos \theta_1 + n_{1c} = a\sqrt{E_c} \cos \theta_1 + n_{1c} \quad (10)$$

and

$$Y_{2c} = n_{2c} \quad (11)$$

where  $E_c = A^2$  is the average energy per diversity bit and

$$n_{1c} = \int_0^T \sqrt{\frac{2}{T}} \cos(2\pi f_1 t) n(t) dt \quad (12)$$

and

$$n_{2c} = \int_0^T \sqrt{\frac{2}{T}} \cos(2\pi f_2 t) n(t) dt \quad (13)$$

The quadrature outputs for the receiver are

$$Y_{1s} = aA \sin \theta_1 + n_{1s} = a\sqrt{E_c} \sin \theta_1 + n_{1s} \quad (14)$$

and

$$Y_{2s} = n_{2s} \quad (15)$$

where

$$n_{1s} = \int_0^T \sqrt{\frac{2}{T}} \sin(2\pi f_1 t) n(t) dt \quad (16)$$

and

$$n_{2s} = \int_0^T \sqrt{\frac{2}{T}} \sin(2\pi f_2 t) n(t) dt \quad (17)$$

All the above random variables  $n_{1c}n_{2c}n_{1s}n_{2s}$  are independent, identically distributed, zero mean, Gaussian random variables with variances  $\sigma_n^2$ . At the two branches we have

$$V_1 = Y_{1c}^2 + Y_{1s}^2 = (a\sqrt{E_c} \cos\theta_1 + n_{1c})^2 + (a\sqrt{E_c} \sin\theta_1 + n_{1s})^2 \quad (18)$$

and

$$V_2 = Y_{2c}^2 + Y_{2s}^2 = n_{2c}^2 + n_{2s}^2 \quad (19)$$

Finally at the output of the BFSK demodulator we have

$$V = V_1 - V_2 \quad (20)$$

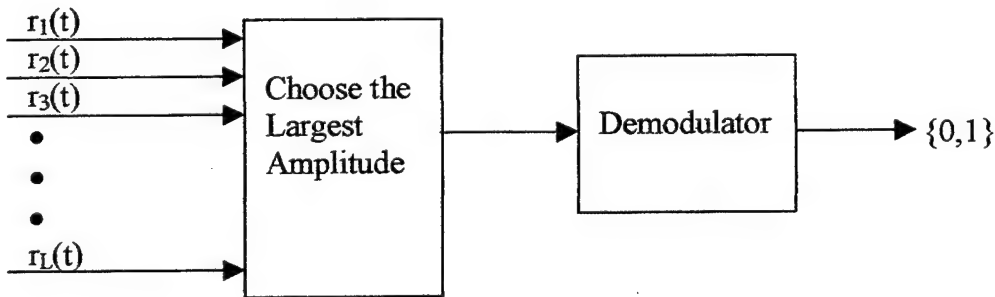


## II. SELECTION COMBINING

Selection Combining (SC) is a diversity combining technique where the signal with the largest amplitude, or largest signal-to-noise ratio, in  $L$  diversity branches is selected. Thus the decision variable for the selection combining technique is defined as

$$\gamma = \max \{ \gamma_1, \gamma_2, \dots, \gamma_L \} \quad , \quad (21)$$

where the signal-to-noise ratios per diversity channel  $\gamma_k$  ( $k = 1, 2, \dots, L$ ) are independent, identically distributed random variables with a Nakagami-  $m$  probability density function as defined in Chapter I. The receiver with selection combining is shown in Fig. 2. From the figure we see that SC is a predetection combining technique.



**Figure 2.** Receiver for selection combining.

### A. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLES

The probability density function of  $\gamma_k$  is given by [1]

$$f_{\Gamma_K}(\gamma_k) = \frac{m^m}{\Gamma(m) \overline{\gamma_c}^m} \gamma_k^{m-1} \exp\left(-\frac{m\gamma_k}{\overline{\gamma_c}}\right) \quad (22)$$

where  $\overline{\gamma_c} = \Omega \frac{E}{N_0}$  (23)

is the average signal-to-noise ratio per diversity channel. Let the average signal-to-noise ratio per bit be  $\overline{\gamma_B}$ , then we have

$$\overline{\gamma_c} = \frac{\overline{\gamma_B}}{L} \quad (24)$$

So the probability density function becomes now

$$f_{\Gamma_K}(\gamma_k) = \frac{m^m L^m}{\Gamma(m) \overline{\gamma_B}^m} \gamma_k^{m-1} \exp\left(-\frac{m\gamma_k L}{\overline{\gamma_B}}\right) \quad (25)$$

The cumulative distribution function (cdf) of the above pdf is

$$F_{\Gamma_K}(\gamma) = \int_0^{\gamma} f_{\Gamma_K}(\gamma_k) d\gamma_k \quad (26)$$

Substituting (25) into (26) we get

$$\begin{aligned}
F_{\Gamma_K}(\gamma) &= \int_0^{\gamma} \frac{m^m L^m}{\Gamma(m) \gamma_B^m} \gamma_k^{m-1} \exp\left(-\frac{m \gamma_k L}{\gamma_B}\right) d\gamma_k \\
&= \frac{m^m L^m}{\Gamma(m) \gamma_B^m} \int_0^{\gamma} \gamma_k^{m-1} \exp\left(-\frac{m \gamma_k L}{\gamma_B}\right) d\gamma_k
\end{aligned} \tag{27}$$

We can eliminate the integral using the following expression [3]

$$\int_0^u x^{v-1} \exp(-\mu x) dx = \mu^{-v} g(v, \mu u) \tag{28}$$

where the real part of  $v$  must be greater than zero. The  $g$ -function is defined as [3]

$$g(a, x) = \int_0^x \exp(-t) t^{a-1} dt \tag{29}$$

where the real part of  $a$  must be greater than zero.

Applying (28) and (29) in (27) we have

$$F_{\Gamma_K}(\gamma) = \frac{m^m L^m}{\Gamma(m) \gamma_B^m} \left(\frac{mL}{\gamma_B}\right)^{-m} g\left(m, \frac{m\gamma L}{\gamma_B}\right) \tag{30}$$

where

$$g\left(m, \frac{m\gamma L}{\gamma_B}\right) = \int_0^{\frac{m\gamma L}{\gamma_B}} \exp(-t) t^{m-1} dt \tag{31}$$

We can simplify (30) as follows

$$F_{\Gamma_K}(\gamma) = \frac{1}{\Gamma(m)} g(m, \frac{m\gamma L}{\gamma_B}) \quad (32)$$

Now we can define the probability density function for the largest signal-to-noise ratio  $\gamma$  in (21) using the expression [9]

$$f_{\Gamma}(\gamma) = L f_{\Gamma_K}(\gamma) [F_{\Gamma_K}(\gamma)]^{L-1} \quad (33)$$

Substituting (25) and (32) into (33) we have

$$f_{\Gamma}(\gamma) = \frac{L^{m+1} m^m}{[\Gamma(m)]^L \gamma_B^m} \gamma^{m-1} \exp(-\frac{m\gamma L}{\gamma_B}) [g(m, \frac{m\gamma L}{\gamma_B})]^{L-1} \quad (34)$$

## B. BIT ERROR PROBABILITY

The bit error rate expression of BFSK for a fading channel, conditioned on the signal-to-noise ratio  $\gamma$ , is given by [1]

$$P_B(\gamma) = \frac{1}{2} \exp(-\frac{\gamma}{2}) \quad (35)$$

In order to obtain the error probability expression of BFSK for the Nakagami-  $m$  fading channel, we use the following integral

$$P_B = \int_0^{\infty} P_B(\gamma) f_{\Gamma}(\gamma) d\gamma \quad (36)$$

Substituting (34) and (35) into (36) we have

$$\begin{aligned}
 P_B &= \int_0^\infty \frac{1}{2} \exp\left(-\frac{\gamma}{2}\right) \frac{L^{m+1} m^m}{[\Gamma(m)]^L \gamma_B^m} \gamma^{m-1} \exp\left(-\frac{m\gamma L}{\gamma_B}\right) \left[g\left(m, \frac{m\gamma L}{\gamma_B}\right)\right]^{L-1} d\gamma \\
 &= \frac{L^{m+1} m^m}{2[\Gamma(m)]^L \gamma_B^m} \int_0^\infty \gamma^{m-1} \exp\left[-\gamma \cdot \left(\frac{1}{2} + \frac{mL}{\gamma_B}\right)\right] \cdot \left[g\left(m, \frac{m\gamma L}{\gamma_B}\right)\right]^{L-1} d\gamma
 \end{aligned} \tag{37}$$



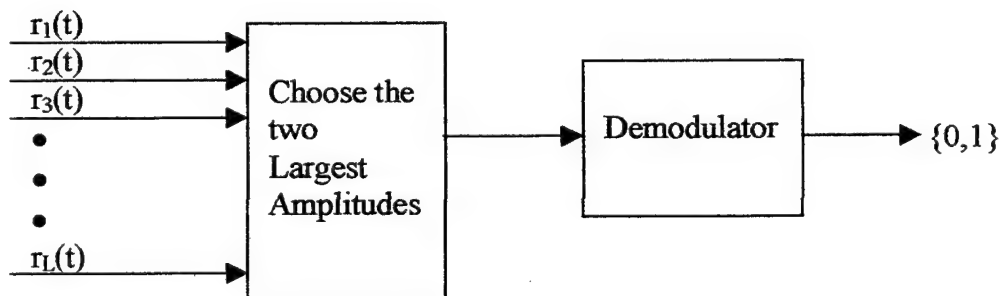


### III. SECOND ORDER SELECTION COMBINING (SC-2)

For second order selection combining (SC-2) we use the two largest amplitudes as decision variables from the  $L$  diversity branches. Thus the two decision variables for SC-2 are defined as

$$\begin{aligned} V_1 &= \max \{ \gamma_1, \gamma_2, \dots, \gamma_L \} \\ V_2 &= \text{second max} \{ \gamma_1, \gamma_2, \dots, \gamma_L \} \end{aligned} \quad , \quad (38)$$

where the signal-to-noise ratios per diversity channel  $\gamma_k$  ( $k = 1, 2, \dots, L$ ) are independent, identically distributed random variables with a Nakagami-  $m$  probability density function as defined in Chapter I. This technique is a predetection combining technique with the receiver shown in Fig. 3.



**Figure 3.** Receiver for the second order selection combining.

### A. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLES

The probability density function of  $\gamma_k$  is given by (25). Since we have two decision variables, we have to define their joint probability density function [9]

$$f_{v_1, v_2}(v_1, v_2) = L(L-1)f_{\Gamma_K}(v_1)f_{\Gamma_K}(v_2)[F_{\Gamma_K}(v_2)]^{L-2} \quad (39)$$

where  $v_1 \geq v_2$ . The selection combiner output  $\gamma$  is

$$\gamma = v_1 + v_2 \quad (40)$$

Now we must derive a probability density function for the random variable  $\gamma$ . The cumulative probability function for the random variable  $\gamma$  is defined as [9]

$$F_{\Gamma}(\gamma) = \int_0^{\gamma/2} \int_{v_2}^{\gamma-v_2} f_{v_1, v_2}(v_1, v_2) dv_1 dv_2 \quad (41)$$

Substituting (39) into (41) we have

$$F_{\Gamma}(\gamma) = \int_0^{\gamma/2} \int_{v_2}^{\gamma-v_2} L(L-1)f_{\Gamma_K}(v_1)f_{\Gamma_K}(v_2)[F_{\Gamma_K}(v_2)]^{L-2} dv_1 dv_2 \quad (42)$$

The cumulative probability function  $F_{\Gamma_K}(v_2)$  is defined in (32) in Chapter II. So substituting (25) and (32) into (42) we get the following expression for the cumulative probability function

$$F_{\Gamma}(\gamma) = \int_0^{\gamma/2} \int_{v_2}^{\gamma-v_2} L(L-1) \frac{m^m L^m}{\Gamma(m) \gamma_B^m} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) \frac{m^m L^m}{\Gamma(m) \gamma_B^m} \\ \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \frac{1}{[\Gamma(m)]^{L-2}} \left[g\left(m, \frac{m v_2 L}{\gamma_B}\right)\right]^{L-2} dv_1 dv_2 \quad (43)$$

Simplifying and separating the integrals we have

$$F_{\Gamma}(\gamma) = L(L-1) \frac{m^{2m} L^{2m}}{[\Gamma(m)]^L \gamma_B^{2m}} \int_0^{\gamma/2} \left\{ \int_{v_2}^{\gamma-v_2} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 \right\} \\ \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \left[g\left(m, \frac{m v_2 L}{\gamma_B}\right)\right]^{L-2} dv_2 \quad (44)$$

The internal integral can be expressed in a simpler way using (29) as

$$\int_{v_2}^{\gamma-v_2} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 = \int_0^{\gamma-v_2} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 - \\ - \int_0^{v_2} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 = \\ = \left(\frac{mL}{\gamma_B}\right)^{-m} \left[g\left(m, \frac{m(\gamma-v_2)L}{\gamma_B}\right) - g\left(m, \frac{m v_2 L}{\gamma_B}\right)\right] \quad (45)$$

Substituting (45) into (44) leads to

$$F_{\Gamma}(\gamma) = L(L-1) \frac{m^m L^m}{[\Gamma(m)]^L \gamma_B^m} \int_0^{\gamma/2} \left[ g\left(m, \frac{m(\gamma-v_2)L}{\gamma_B}\right) - g\left(m, \frac{m v_2 L}{\gamma_B}\right) \right] \\ \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \left[g\left(m, \frac{m v_2 L}{\gamma_B}\right)\right]^{L-2} dv_2 \quad (46)$$

The probability density function corresponding to the above cummulative density function is obtained as follows

$$f_{\Gamma}(\gamma) = \frac{d}{d\gamma}[F_{\Gamma}(\gamma)] \quad (47)$$

Substituting (46) into (47) we have

$$\begin{aligned} f_{\Gamma}(\gamma) &= L(L-1) \frac{m^m L^m}{[\Gamma(m)]^L \gamma_B^m} \frac{d}{d\gamma} \left\{ \int_0^{\gamma/2} \left[ g\left(m, \frac{m(\gamma - v_2)L}{\gamma_B}\right) - g\left(m, \frac{m v_2 L}{\gamma_B}\right) \right] \right. \\ &\quad \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \left[ g\left(m, \frac{m v_2 L}{\gamma_B}\right) \right]^{L-2} dv_2 \Big\} = \\ &= L(L-1) \frac{m^m L^m}{[\Gamma(m)]^L \gamma_B^m} \left\{ \frac{d}{d\gamma} \left[ \int_0^{\gamma/2} \left[ g\left(m, \frac{m(\gamma - v_2)L}{\gamma_B}\right) \right] \left[ g\left(m, \frac{m v_2 L}{\gamma_B}\right) \right]^{L-2} \right. \right. \\ &\quad \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 \Big] - \frac{d}{d\gamma} \left[ \int_0^{\gamma/2} \left[ g\left(m, \frac{m v_2 L}{\gamma_B}\right) \right]^{L-1} v_2^{m-1} \right. \\ &\quad \times \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 \Big] \Big\} \quad (48) \end{aligned}$$

Now we use Leibnitz's rule, given by

$$\frac{d \left[ \int_{a(x)}^{b(x)} f(\lambda, x) d\lambda \right]}{dx} = f(b(x), x) \frac{db(x)}{dx} - f(a(x), x) \frac{da(x)}{dx} + \int_{a(x)}^{b(x)} \frac{df(\lambda, x)}{dx} d\lambda \quad (49)$$

Using (49) expression (48) becomes

$$\begin{aligned} f_{\Gamma}(\gamma) &= L(L-1) \frac{m^m L^m}{[\Gamma(m)]^L \gamma_B^m} \left\{ \frac{1}{2} \left[ g\left(m, \frac{m \gamma L}{2 \gamma_B}\right) \right]^{L-1} \left(\frac{\gamma}{2}\right)^{m-1} \right. \\ &\quad \times \exp\left(-\frac{m \gamma L}{2 \gamma_B}\right) - 0 + \int_0^{\gamma/2} v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \left[ g\left(m, \frac{m v_2 L}{\gamma_B}\right) \right]^{L-2} \\ &\quad \times \frac{d}{d\gamma} \left( g\left(m, \frac{m(\gamma - v_2)L}{\gamma_B}\right) \right) dv_2 - \frac{1}{2} \left[ g\left(m, \frac{m \gamma L}{2 \gamma_B}\right) \right]^{L-1} \left(\frac{\gamma}{2}\right)^{m-1} \\ &\quad \times \exp\left(-\frac{m \gamma L}{2 \gamma_B}\right) + 0 - 0 \Big\} = \end{aligned}$$

$$\begin{aligned}
&= L(L-1) \frac{m^m L^m}{[\Gamma(m)]^L \gamma_B^m} \int_0^{\gamma/2} v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \left[g\left(m, \frac{m v_2 L}{\gamma_B}\right)\right]^{L-2} \\
&\quad \times \frac{d}{d\gamma} \left(g\left(m, \frac{m(\gamma - v_2)L}{\gamma_B}\right)\right) dv_2
\end{aligned} \tag{50}$$

Now employing again the Leibnitz's rule at the last derivative and using (28) we have

$$\begin{aligned}
\frac{d}{d\gamma} \left(g\left(m, \frac{m(\gamma - v_2)L}{\gamma_B}\right)\right) &= \frac{d}{d\gamma} \left( \int_0^{\frac{m(\gamma - v_2)L}{\gamma_B}} \exp(-t) t^{m-1} dt \right) = \\
&= \frac{m L}{\gamma_B} \left[ \exp\left(-\frac{m(\gamma - v_2)L}{\gamma_B}\right) \right] \left(\frac{m(\gamma - v_2)L}{\gamma_B}\right)^{m-1} - 0 + 0 \\
&= \frac{m L}{\gamma_B} \left[ \exp\left(-\frac{m(\gamma - v_2)L}{\gamma_B}\right) \right] \left(\frac{m(\gamma - v_2)L}{\gamma_B}\right)^{m-1}
\end{aligned} \tag{51}$$

Using (51), the expression (50) becomes

$$\begin{aligned}
f_{\Gamma}(\gamma) &= (L-1) \frac{m^{2m} L^{2m+1}}{[\Gamma(m)]^L \gamma_B^{2m}} \exp\left(-\frac{m\gamma L}{\gamma_B}\right) \int_0^{\gamma/2} (v_2(\gamma - v_2))^{m-1} \\
&\quad \times \left[g\left(m, \frac{m v_2 L}{\gamma_B}\right)\right]^{L-2} dv_2
\end{aligned} \tag{52}$$

## B. BIT ERROR PROBABILITY

The bit error rate expression of BFSK with second order selection combining,

conditioning on the signal-to-noise ratio  $\gamma$  in (40), is given by [9]

$$P_B(\gamma) = \frac{1}{8}[\exp(-\frac{\gamma}{2})](4 + \frac{\gamma}{2}) \quad (53)$$

In order to obtain the error probability expression of BFSK for the Nakagami-  $m$  fading channel with SC-2 technique, we use the following integral

$$P_B = \int_0^{\infty} P_B(\gamma) f_{\Gamma}(\gamma) d\gamma \quad (54)$$

Substituting (52) and (53) into (54) we have

$$P_B = (L-1) \frac{m^{2m} L^{2m+1}}{8 [\Gamma(m)]^L \gamma_B^{2m}} \int_0^{\infty} (4 + \frac{\gamma}{2}) \exp[-\gamma(\frac{1}{2} + \frac{mL}{\gamma_B})] \\ \times \left\{ \int_0^{\gamma/2} (v_2(\gamma - v_2))^{m-1} [g(m, \frac{mv_2 L}{\gamma_B})]^{L-2} dv_2 \right\} d\gamma \quad (55)$$

#### IV. THIRD ORDER SELECTION COMBINING (SC-3)

For third order Selection Combining (SC-3) we use the three largest amplitudes as decision variables from the  $L$  diversity branches. From  $L$  independent, identically distributed random variables, the three decision variables for SC-3 are defined as

$$\begin{aligned} V_1 &= \max \{\gamma_1, \gamma_2, \dots, \gamma_L\} \\ V_2 &= \text{second max} \{\gamma_1, \gamma_2, \dots, \gamma_L\} \\ V_3 &= \text{third max} \{\gamma_1, \gamma_2, \dots, \gamma_L\} \end{aligned} \quad (56)$$

where  $\gamma_k$  ( $k = 1, 2, \dots, L$ ) are independent, identically distributed random variables with a Nakagami- $m$  probability density function as is defined in Chapter I. Again this technique is a predetection combining technique and the respective receiver is shown in Fig. 4.

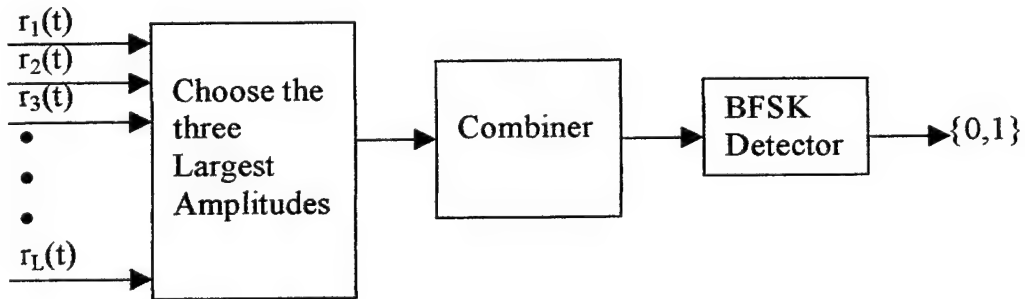


Figure 4. Receiver for the third order Selection Combining



### A. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLES

The probability density function for each  $\gamma_k$  is given by (25). Since we have three decision variables, we have to define their joint probability density function [9]

$$f_{v_1, v_2, v_3}(v_1, v_2, v_3) = L(L-1)(L-2)f_{\Gamma_K}(v_3)f_{\Gamma_K}(v_2)f_{\Gamma_K}(v_1) \times [F_{\Gamma_K}(v_3)]^{L-3} \quad (57)$$

where  $v_1 \geq v_2 \geq v_3$ . Now since the selection combiner output  $\gamma$  is

$$\gamma = v_1 + v_2 + v_3 \quad (58)$$

we have to derive a probability density function for the random variable  $\gamma$ . The cumulative density function for  $\gamma$  is defined as [9]

$$F_{\Gamma}(\gamma) = \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} \int_{v_2}^{\gamma-v_2-v_3} f_{v_1, v_2, v_3}(v_1, v_2, v_3) dv_1 dv_2 dv_3 \quad (59)$$

Substituting (57) into (59) we have

$$F_{\Gamma}(\gamma) = \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} \int_{v_2}^{\gamma-v_2-v_3} L(L-1)(L-2)f_{\Gamma_K}(v_3)f_{\Gamma_K}(v_2)f_{\Gamma_K}(v_1) \times [F_{\Gamma_K}(v_3)]^{L-3} dv_1 dv_2 dv_3 \quad (60)$$

Substituting (25) and (32) into (60) we have

$$\begin{aligned}
F_{\Gamma}(\gamma) &= \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} \int_{v_2}^{\gamma-v_2-v_3} L(L-1)(L-2) \frac{m^m L^m}{\Gamma(m) \gamma_B^m} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) \\
&\times \frac{m^m L^m}{\Gamma(m) \gamma_B^m} v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \frac{m^m L^m}{\Gamma(m) \gamma_B^m} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) \\
&\times \frac{1}{[\Gamma(m)]^{L-3}} \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} dv_1 dv_2 dv_3
\end{aligned} \tag{61}$$

Simplifying and separating the inner integral we get

$$\begin{aligned}
F_{\Gamma}(\gamma) &= L(L-1)(L-2) \frac{m^{3m} L^{3m}}{[\Gamma(m)]^L \gamma_B^{3m}} \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} \left\{ \int_{v_2}^{\gamma-v_2-v_3} v_1^{m-1} \right. \\
&\times \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 \Big\} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \\
&\times \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} dv_2 dv_3
\end{aligned} \tag{62}$$

Using (28) we can eliminate the inner integral as follows

$$\begin{aligned}
\int_{v_2}^{\gamma-v_2-v_3} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 &= \int_0^{\gamma-v_2-v_3} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 - \\
&- \int_0^{v_2} v_1^{m-1} \exp\left(-\frac{m v_1 L}{\gamma_B}\right) dv_1 = \\
&= \left(\frac{m L}{\gamma_B}\right)^{-m} \left[g\left(m, \frac{m(\gamma-v_2-v_3)L}{\gamma_B}\right) - g\left(m, \frac{m v_2 L}{\gamma_B}\right)\right]
\end{aligned} \tag{63}$$

So using (63) the expression (62) becomes

$$\begin{aligned}
F_{\Gamma}(\gamma) = & L(L-1)(L-2) \frac{m^{2m} L^{2m}}{[\Gamma(m)]^L \gamma_B^{2m}} \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} \left[ g\left(m, \frac{m(\gamma - v_2 - v_3)L}{\gamma_B}\right) \right. \\
& - g\left(m, \frac{m v_2 L}{\gamma_B}\right) \left. \right] v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \\
& \times \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} dv_2 dv_3
\end{aligned} \tag{64}$$

The probability density function for the random variable  $\gamma$  is given by (47)

$$\begin{aligned}
f_{\Gamma}(\gamma) = & L(L-1)(L-2) \frac{m^{2m} L^{2m}}{[\Gamma(m)]^L \gamma_B^{2m}} \frac{d}{d\gamma} \left\{ \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} \left[ g\left(m, \frac{m(\gamma - v_2 - v_3)L}{\gamma_B}\right) \right. \right. \\
& - g\left(m, \frac{m v_2 L}{\gamma_B}\right) \left. \right] v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \\
& \times \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} dv_2 dv_3 \left. \right\}
\end{aligned} \tag{65}$$

Separating the derivative into two terms results in

$$\begin{aligned}
f_{\Gamma}(\gamma) = & L(L-1)(L-2) \frac{m^{2m} L^{2m}}{[\Gamma(m)]^L \gamma_B^{2m}} \left[ \frac{d}{d\gamma} \left\{ \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} g\left(m, \frac{m(\gamma - v_2 - v_3)L}{\gamma_B}\right) \right. \right. \\
& \times \left. \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 dv_3 \right\} \right. \\
& - \frac{d}{d\gamma} \left\{ \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} g\left(m, \frac{m v_2 L}{\gamma_B}\right) \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) \right. \\
& \times \left. v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 dv_3 \right\} \left. \right]
\end{aligned} \tag{66}$$

We can now employ Leibnitz's rule (49) separately to the two terms. The first term becomes

$$\begin{aligned}
& \frac{d}{d\gamma} \left\{ \int_0^{\frac{\gamma}{2}} \int_{v_3}^{\frac{\gamma-v_3}{2}} g\left(m, \frac{m(\gamma-v_2-v_3)L}{\gamma_B}\right) \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) \right. \\
& \quad \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 dv_3 \left. \right\} = \\
& = 0 - 0 + \int_0^{\frac{\gamma}{2}} \frac{d}{d\gamma} \left\{ \int_{v_3}^{\frac{\gamma-v_3}{2}} g\left(m, \frac{m(\gamma-v_2-v_3)L}{\gamma_B}\right) v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 \right\} \\
& \quad \times \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) dv_3 = \\
& = \int_0^{\frac{\gamma}{2}} \left[ \frac{1}{2} g\left(m, \frac{m(\gamma-v_3)L}{2\gamma_B}\right) \exp\left(-\frac{m(\gamma-v_3)L}{2\gamma_B}\right) \left(\frac{\gamma-v_3}{2}\right)^{m-1} \right. \\
& \quad \times \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) - 0 + v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) \\
& \quad \times \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} \int_{v_3}^{\frac{\gamma-v_3}{2}} v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) \\
& \quad \times \frac{d}{d\gamma} \left\{ g\left(m, \frac{m(\gamma-v_2-v_3)L}{\gamma_B}\right) \left. \right\} dv_2 \right] dv_3 .
\end{aligned} \tag{67}$$

Again employing Leibnitz's rule and using (29) we can eliminate the last derivative as follows

$$\begin{aligned}
& \frac{d}{d\gamma} \left\{ g\left(m, \frac{m(\gamma-v_2-v_3)L}{\gamma_B}\right) \right\} = \frac{d}{d\gamma} \left\{ \int_0^{\frac{\gamma_B}{\gamma}} \exp(-t) t^{m-1} dt \right\} \\
& = \left(\frac{mL}{\gamma_B}\right)^m \left[ \exp\left(-\frac{m(\gamma-v_2-v_3)L}{\gamma_B}\right) (\gamma-v_2-v_3)^{m-1} - 0 + 0 \right] .
\end{aligned} \tag{68}$$

So the first term finally becomes

$$\begin{aligned}
& \frac{d}{d\gamma} \left\{ \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} g\left(m, \frac{m(\gamma - v_2 - v_3)L}{\gamma_B}\right) \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) \right. \\
& \quad \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 dv_3 \left. \right\} = \\
& = \int_0^{\frac{\gamma}{3}} \left[ \frac{1}{2} g\left(m, \frac{m(\gamma - v_3)L}{2\gamma_B}\right) \exp\left(-\frac{m(\gamma - v_3)L}{2\gamma_B}\right) \left(\frac{\gamma - v_3}{2}\right)^{m-1} \right. \\
& \quad \times \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) + v_3^{m-1} \left(\frac{mL}{\gamma_B}\right)^m \\
& \quad \times \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) \int_{v_3}^{\frac{\gamma-v_3}{2}} v_2^{m-1} (\gamma - v_2 - v_3)^{m-1} dv_2 \left. \right] dv_3 \quad (69)
\end{aligned}$$

The second term becomes

$$\begin{aligned}
& \frac{d}{d\gamma} \left\{ \int_0^{\frac{\gamma}{3}} \int_{v_3}^{\frac{\gamma-v_3}{2}} g\left(m, \frac{m v_2 L}{\gamma_B}\right) \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) \right. \\
& \quad \times v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 dv_3 \left. \right\} = \\
& = 0 - 0 + \int_0^{\frac{\gamma}{3}} \frac{d}{d\gamma} \left\{ \int_{v_3}^{\frac{\gamma-v_3}{2}} g\left(m, \frac{m v_2 L}{\gamma_B}\right) v_2^{m-1} \exp\left(-\frac{m v_2 L}{\gamma_B}\right) dv_2 \right\} \\
& \quad \times \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) dv_3 = \\
& = \int_0^{\frac{\gamma}{3}} \frac{1}{2} g\left(m, \frac{m(\gamma - v_3)L}{2\gamma_B}\right) \exp\left(-\frac{m(\gamma - v_3)L}{2\gamma_B}\right) \left(\frac{\gamma - v_3}{2}\right)^{m-1} \\
& \quad \times \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} v_3^{m-1} \exp\left(-\frac{m v_3 L}{\gamma_B}\right) dv_3 - 0 + 0 \quad (70)
\end{aligned}$$

Substituting (69) and (70) into (66) we get

$$\begin{aligned}
f_{\Gamma}(\gamma) &= L(L-1)(L-2) \frac{m^{2m} L^{2m}}{[\Gamma(m)]^L \gamma_B^{2m}} \left\{ \int_0^{\frac{\gamma}{2}} \frac{1}{2} g\left(m, \frac{m(\gamma - v_3)L}{2\gamma_B}\right) \right. \\
&\quad \times \exp\left(-\frac{m(\gamma - v_3)L}{2\gamma_B}\right) \left(\frac{\gamma - v_3}{2}\right)^{m-1} \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} v_3^{m-1} \\
&\quad \times \exp\left(-\frac{m v_3 L}{\gamma_B}\right) + v_3^{m-1} \left(\frac{mL}{\gamma_B}\right)^m \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} \exp\left(-\frac{m \gamma L}{\gamma_B}\right) \\
&\quad \times \int_{v_3}^{\frac{\gamma - v_3}{2}} v_2^{m-1} (\gamma - v_2 - v_3)^{m-1} dv_2 \Big] dv_3 - \int_0^{\frac{\gamma}{3}} \frac{1}{2} g\left(m, \frac{m(\gamma - v_3)L}{2\gamma_B}\right) \\
&\quad \times \exp\left(-\frac{m(\gamma - v_3)L}{2\gamma_B}\right) \left(\frac{\gamma - v_3}{2}\right)^{m-1} \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} v_3^{m-1} \\
&\quad \times \exp\left(-\frac{m v_3 L}{\gamma_B}\right) dv_3 \Big\} = \\
&= (L-1)(L-2) \frac{m^{3m} L^{3m+1}}{[\Gamma(m)]^L \gamma_B^{3m}} \exp\left(-\frac{m \gamma L}{\gamma_B}\right) \int_0^{\frac{\gamma}{3}} \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} \\
&\quad \times v_3^{m-1} \int_{v_3}^{\frac{\gamma - v_3}{2}} v_2^{m-1} (\gamma - v_2 - v_3)^{m-1} dv_2 dv_3
\end{aligned} \tag{71}$$

The probability density function for the random variable  $\gamma$  is given by

$$\begin{aligned}
f_{\Gamma}(\gamma) &= (L-1)(L-2) \frac{m^{3m} L^{3m+1}}{[\Gamma(m)]^L \gamma_B^{3m}} \exp\left(-\frac{m \gamma L}{\gamma_B}\right) \int_0^{\frac{\gamma}{3}} \left[g\left(m, \frac{m v_3 L}{\gamma_B}\right)\right]^{L-3} \\
&\quad \times v_3^{m-1} \int_{v_3}^{\frac{\gamma - v_3}{2}} v_2^{m-1} (\gamma - v_2 - v_3)^{m-1} dv_2 dv_3
\end{aligned} \tag{72}$$

## B. BIT ERROR PROBABILITY

The bit error rate expression of BFSK for the Third Order Selection Combining, conditioned on the Signal-to-Noise ratio, is given by [9]

$$P_B(\gamma) = \frac{1}{32} \left[ \exp\left(-\frac{\gamma}{2}\right) \right] \left( 16 + 3\gamma + \frac{\gamma^2}{8} \right) \quad (73)$$

In order to obtain the error probability expression of BFSK for the Nakagami-  $m$  fading channel for the SC-3 technique, we use the following integral

$$P_B = \int_0^{\infty} P_B(\gamma) f_{\Gamma}(\gamma) d\gamma \quad (74)$$

Substituting (72) and (73) into (74) we have

$$\begin{aligned} P_b &= \int_0^{\infty} \frac{1}{32} \exp\left(-\frac{\gamma}{2}\right) \left( 16 + 3\gamma + \frac{\gamma^2}{8} \right) (L-1)(L-2) \frac{m^{3m} L^{3m+1}}{[\Gamma(m)]^L \gamma_B^{3m}} \\ &\quad \times \exp\left(-\frac{m\gamma L}{\gamma_B}\right) \left\{ \int_0^{\frac{\gamma}{3}} \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} \times v_3^{m-1} \int_{v_3}^{\frac{\gamma-v_3}{2}} v_2^{m-1} \right. \\ &\quad \times (\gamma - v_2 - v_3)^{m-1} dv_2 dv_3 \left. \right\} d\gamma = \\ &= (L-1)(L-2) \frac{m^{3m} L^{3m+1}}{32 [\Gamma(m)]^L \gamma_B^{3m}} \int_0^{\infty} \left( 16 + 3\gamma + \frac{\gamma^2}{8} \right) \\ &\quad \times \exp\left[-\gamma \left( \frac{1}{2} + \frac{m L}{\gamma_B} \right)\right] \left\{ \int_0^{\frac{\gamma}{3}} \left[ g\left(m, \frac{m v_3 L}{\gamma_B}\right) \right]^{L-3} \times v_3^{m-1} \int_{v_3}^{\frac{\gamma-v_3}{2}} v_2^{m-1} \right. \\ &\quad \times (\gamma - v_2 - v_3)^{m-1} dv_2 dv_3 \left. \right\} d\gamma \end{aligned} \quad (75)$$

## V. NUMERICAL RESULTS

In chapters II, III and IV we evaluated the expressions for the bit error rate for noncoherent BFSK signals over a frequency non-selective, slowly fading Nakagami- $m$  channel using SC, SC-2 and SC-3. In order to illustrate the performance and to compare the three techniques, we used MATLAB 5.1 [7] and MATHCAD 7 [8]. The numerical results are shown in Figures 5-32. The bit signal-to-noise ratio is selected in the range of 6-20 dB. Values of the factor  $m=0.5, 0.75, 1, 1.5, 2, 3$  of the Nakagami- $m$  fading channel were used to provide sufficient detail of the performance.

In Figs. 5-10, the first order Selection Combining (SC) performances are presented for diversity orders of  $L=1, 2, 3, 4, 5$  and for the values of  $m$  given above.

In Figs. 11-16, the second order Selection Combining (SC-2) performances are illustrated for the same values of  $m$  as in Figs. 5-10 and for diversity orders of  $L=2, 3, 4, 5$ .

In Figs. 17-22, the third order Selection Combining (SC-3) performances are shown for the same values of  $m$  as in Figs. 5-10 and for diversity orders of  $L=3, 4, 5$ .

In Figs. 23-28, the SC, SC-2 and SC-3 performances are presented for each one of the above values of  $m$  using an arbitrary value for the diversity order  $L$ .

In Figs. 29-31, the SC, SC-2 and SC-3 performances are illustrated separately for all previously chosen values of  $m$  using values of 2, 3 and 4 for the diversity order.

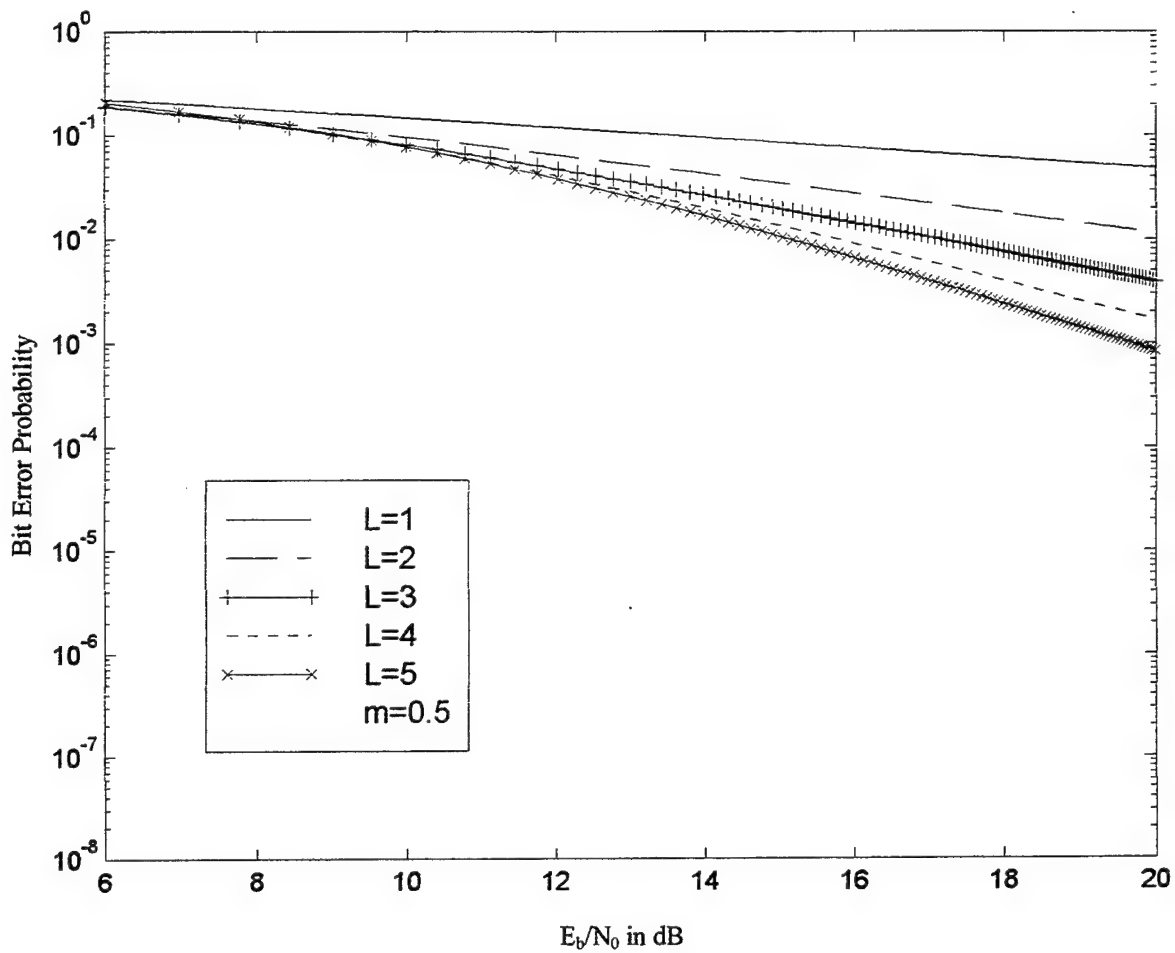
In Figs. 5-22, we note that as  $L$  increases we have a better receiver performance. As  $m$  increases we notice that the system performance with smaller diversity order  $L$  seems to be better than those with larger diversity order  $L$  for low values of the signal-to-noise ratio.



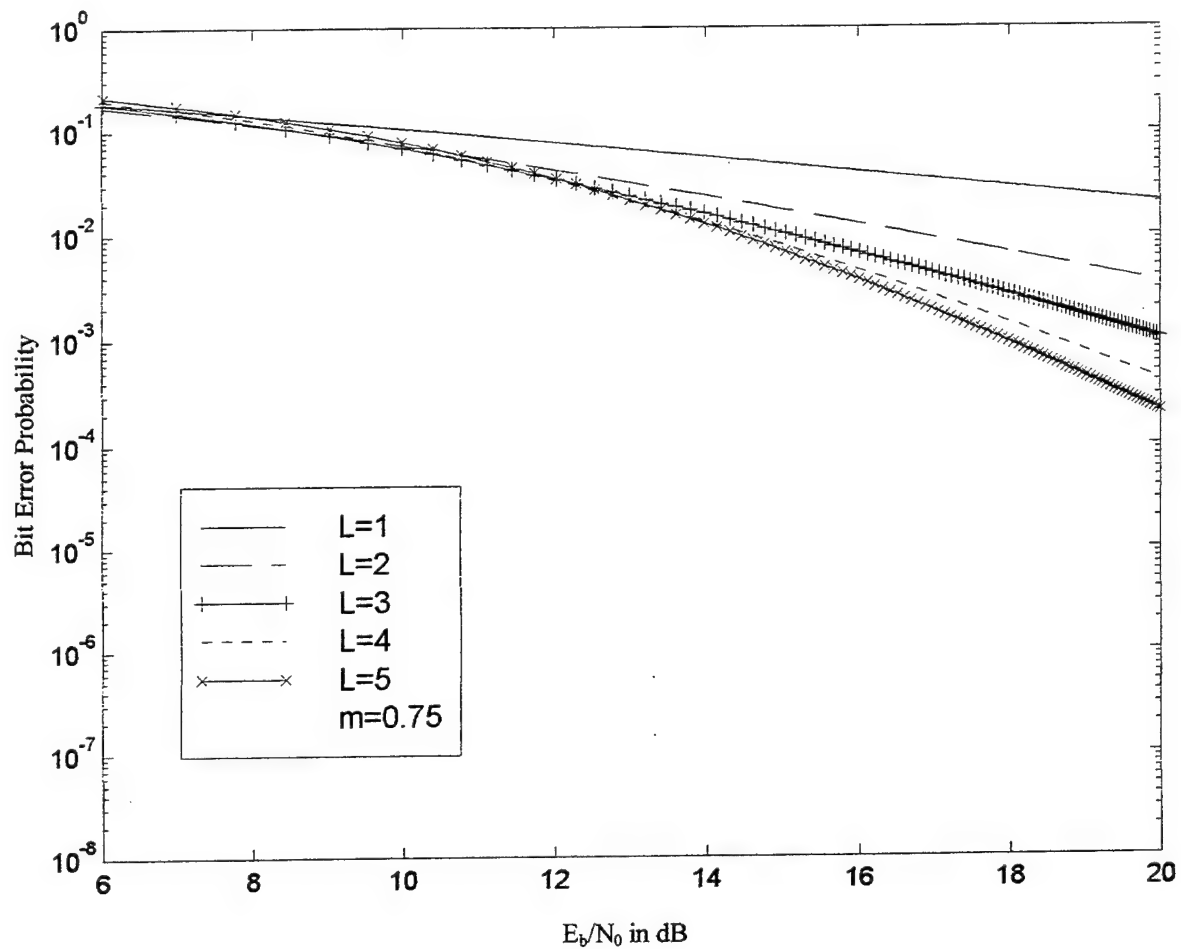
As the signal-to-noise ratio increases this phenomenon becomes reversed. This happens because of the noncoherent combining loss. So in conclusion, a system with a higher diversity order  $L$  performs better than a system with a smaller  $L$ .

In Figs. 23-28, we can clearly notice that for any value of  $L$  or  $m$  we choose, the SC-3 technique performs better than the other two techniques and also the SC-2 technique is superior to the SC technique. We can also see that as the order of diversity or the factor  $m$  increases the performance differences between the three techniques increase in favor of the greater order technique. So in conclusion, as the order increases the Selection Combining techniques correspondingly perform better.

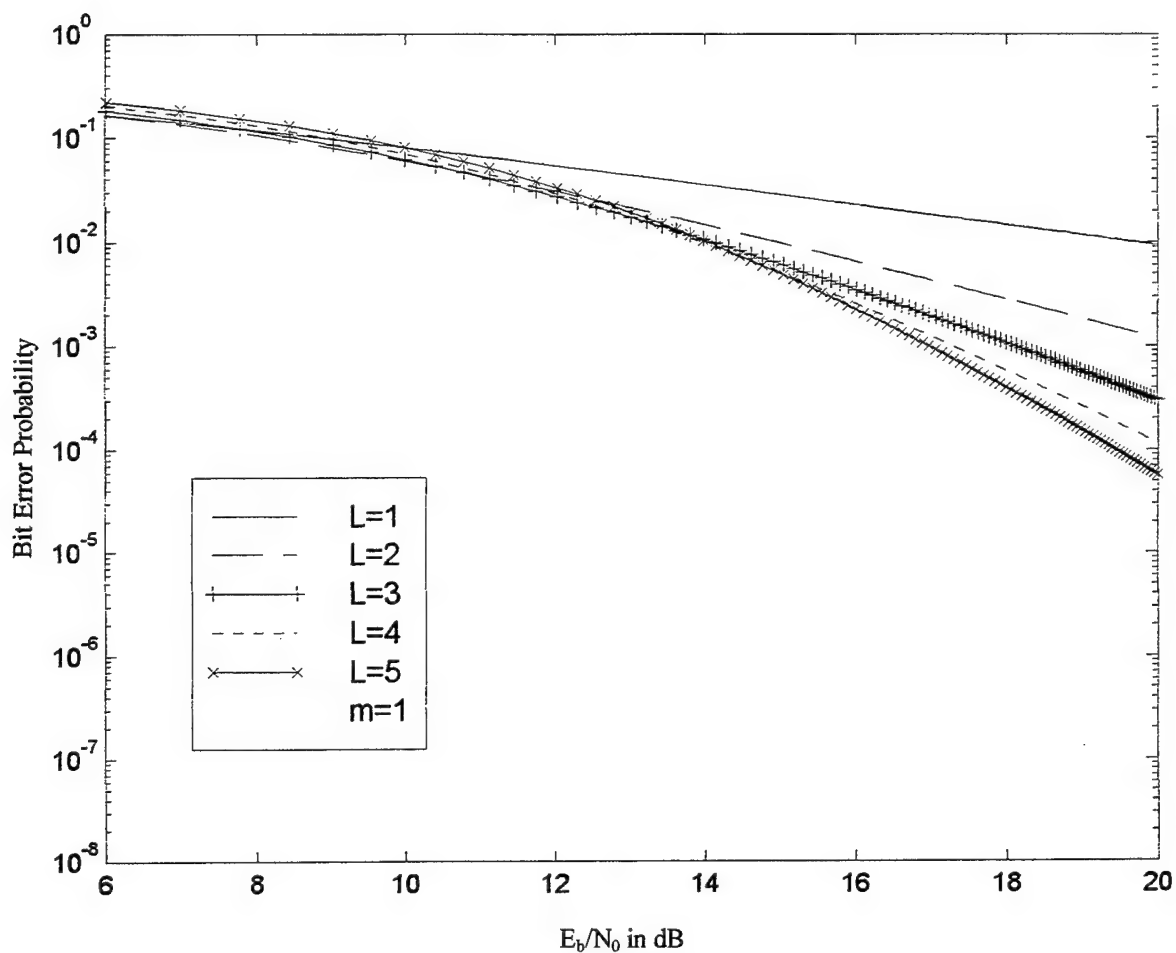
In Figs. 29-31, we can observe that as the factor  $m$  increases the system performs better. This happens for all the techniques and is something we expected since as  $m$  tends to infinity the channel becomes non-fading.



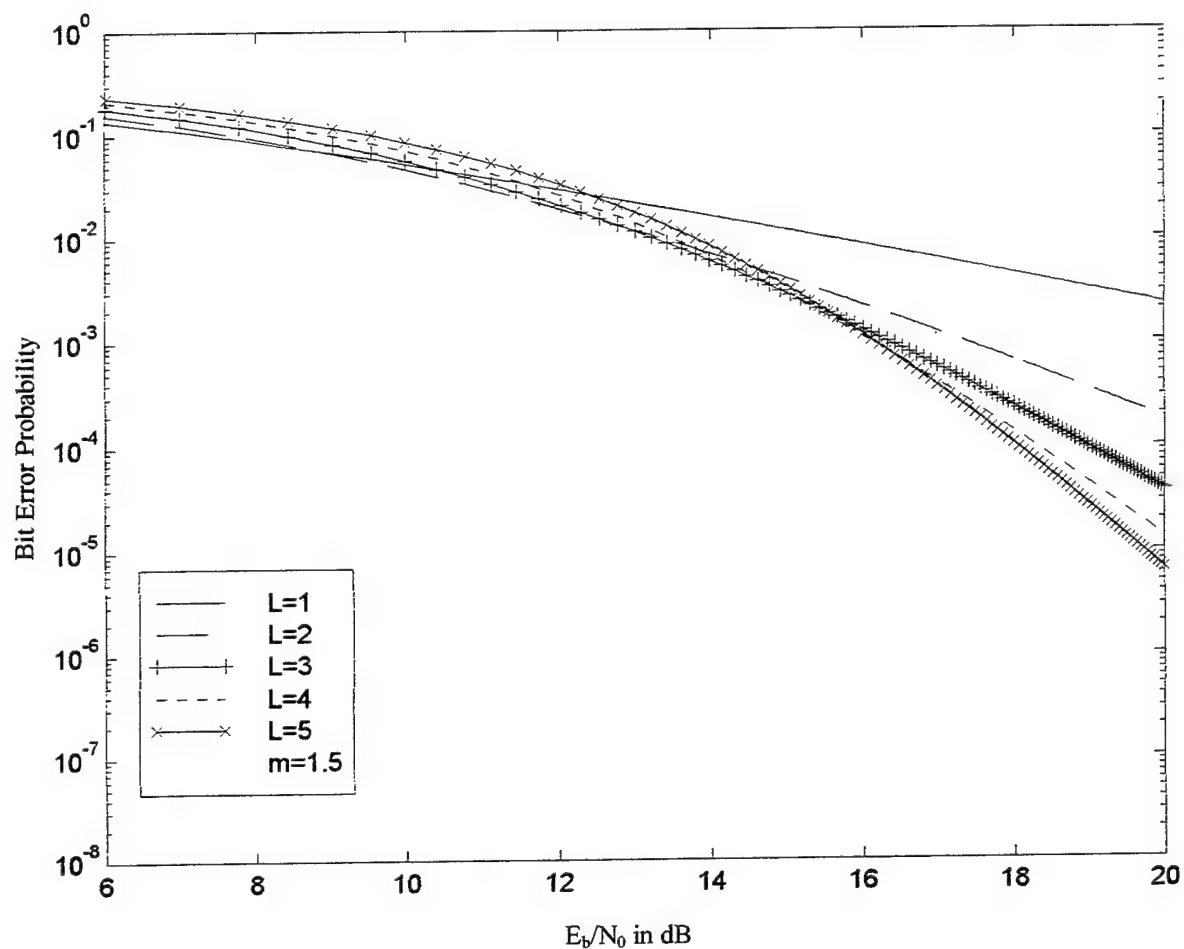
**Figure 5.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.5$ , using first order Selection Combining (SC) for diversity orders of  $L = 1, 2, 3, 4$  and  $5$ .



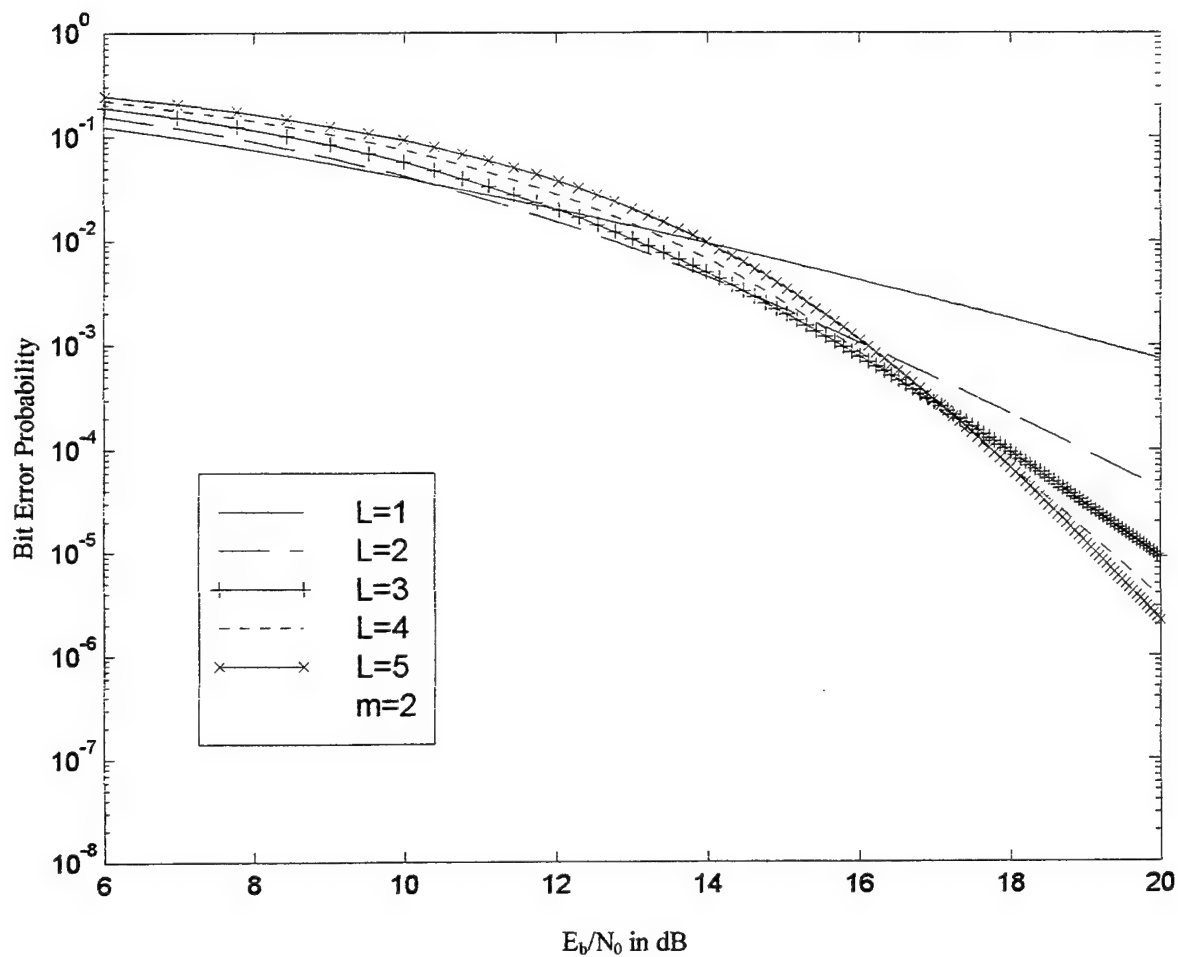
**Figure 6.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.75$ , using first order Selection Combining (SC) for diversity orders of  $L = 1, 2, 3, 4$  and  $5$ .



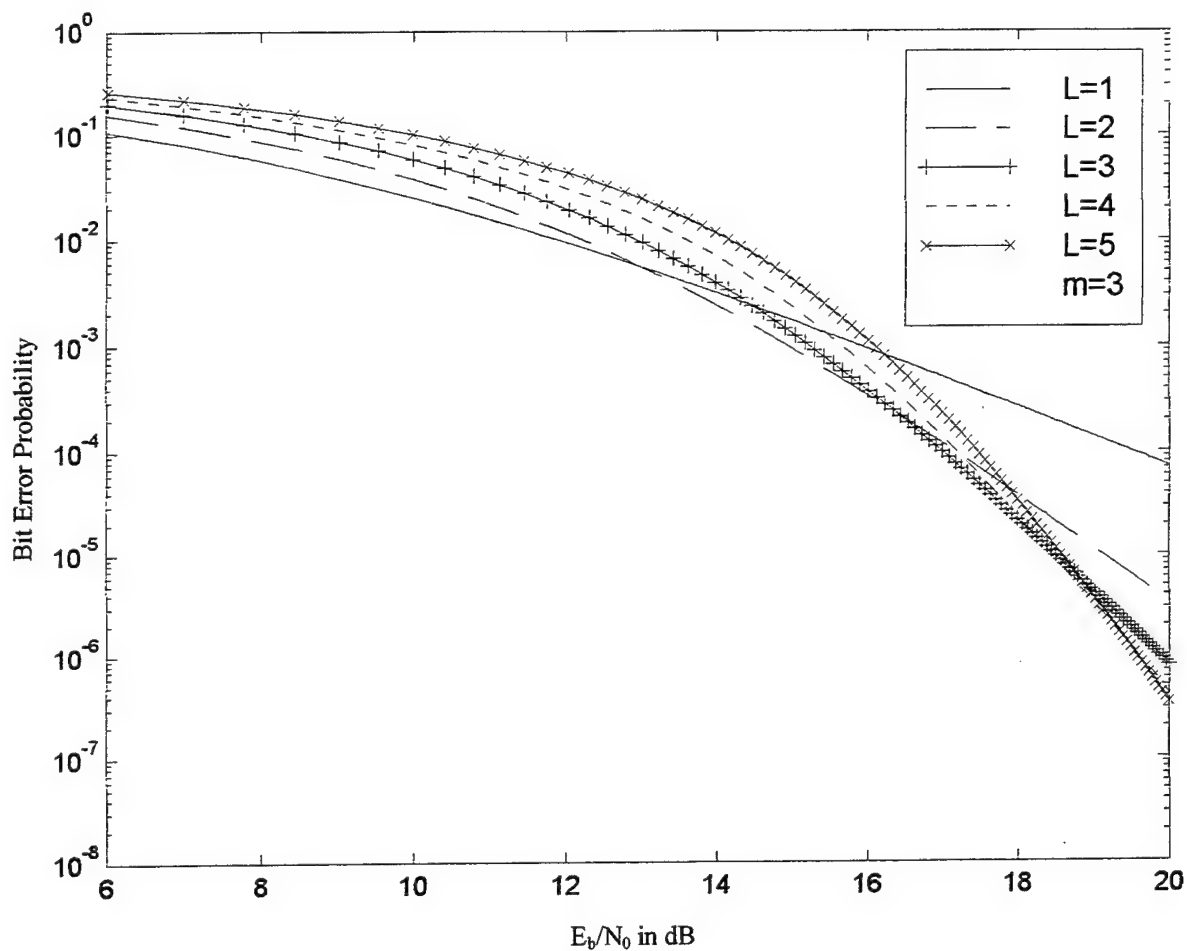
**Figure 7.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 1$ , using first order Selection Combining (SC) for diversity orders of  $L = 1, 2, 3, 4$  and  $5$ .



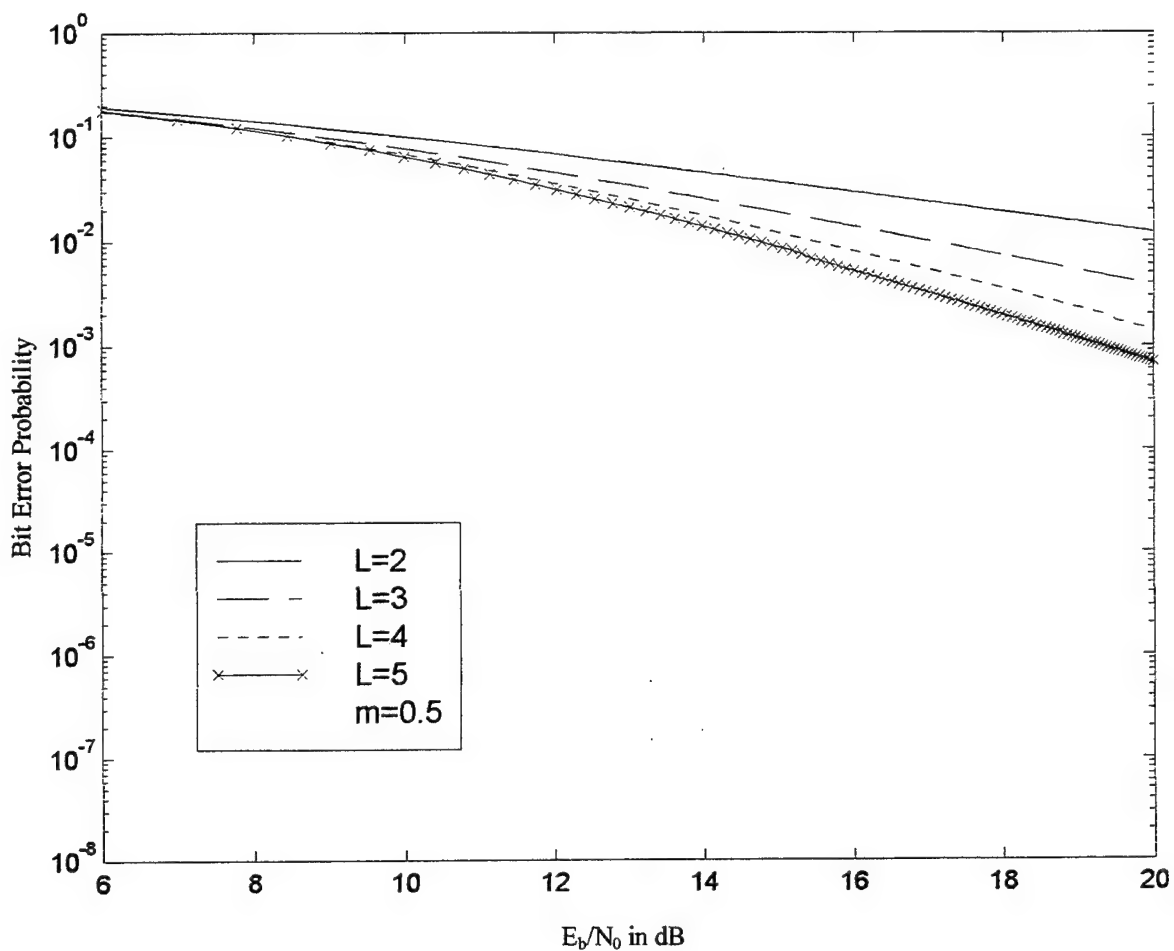
**Figure 8.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 1.5$ , using first order Selection Combining (SC) for diversity orders of  $L = 1, 2, 3, 4$  and  $5$ .



**Figure 9.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 2$ , using first order Selection Combining (SC) for diversity orders of  $L = 1, 2, 3, 4$  and  $5$ .

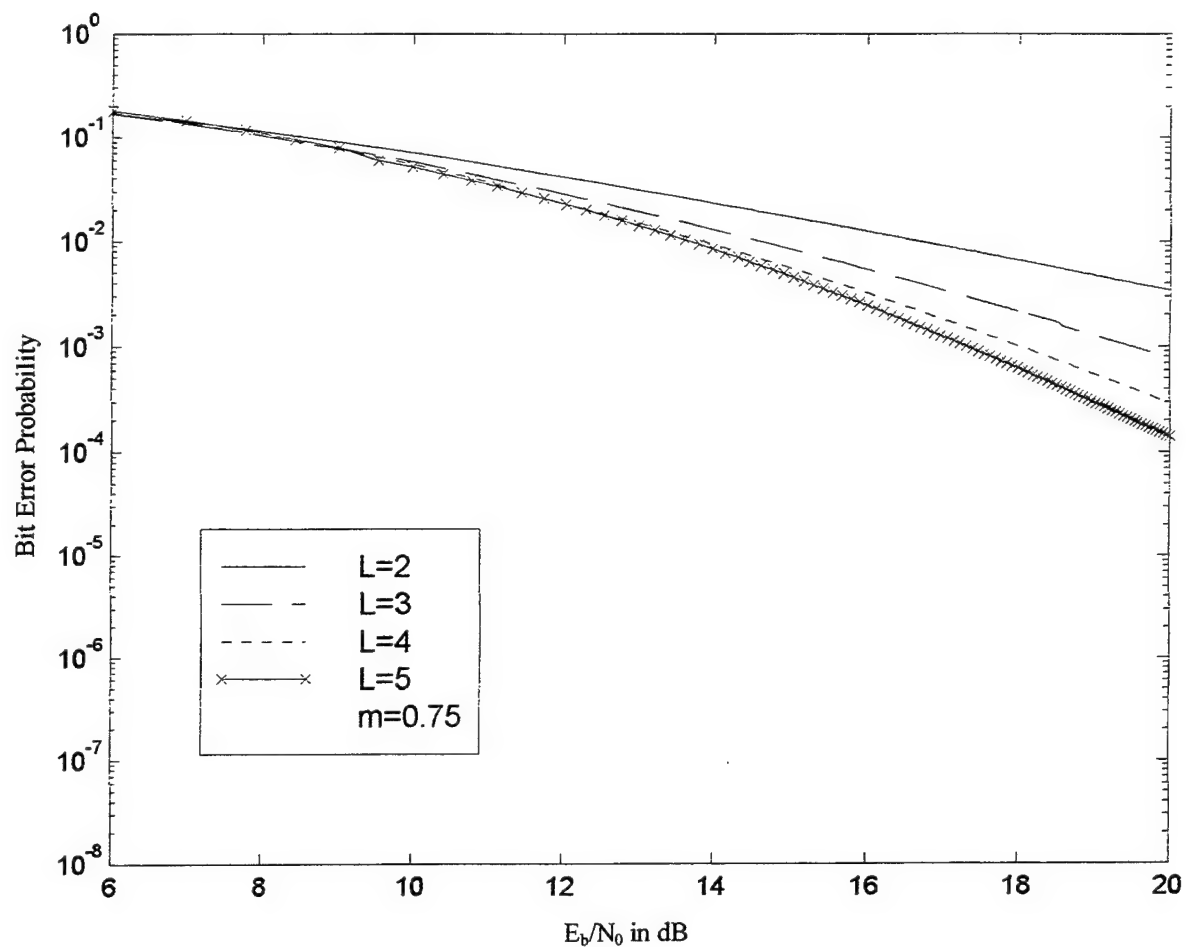


**Figure 10.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 3$ , using first order Selection Combining (SC) for diversity orders of  $L = 1, 2, 3, 4$  and  $5$ .

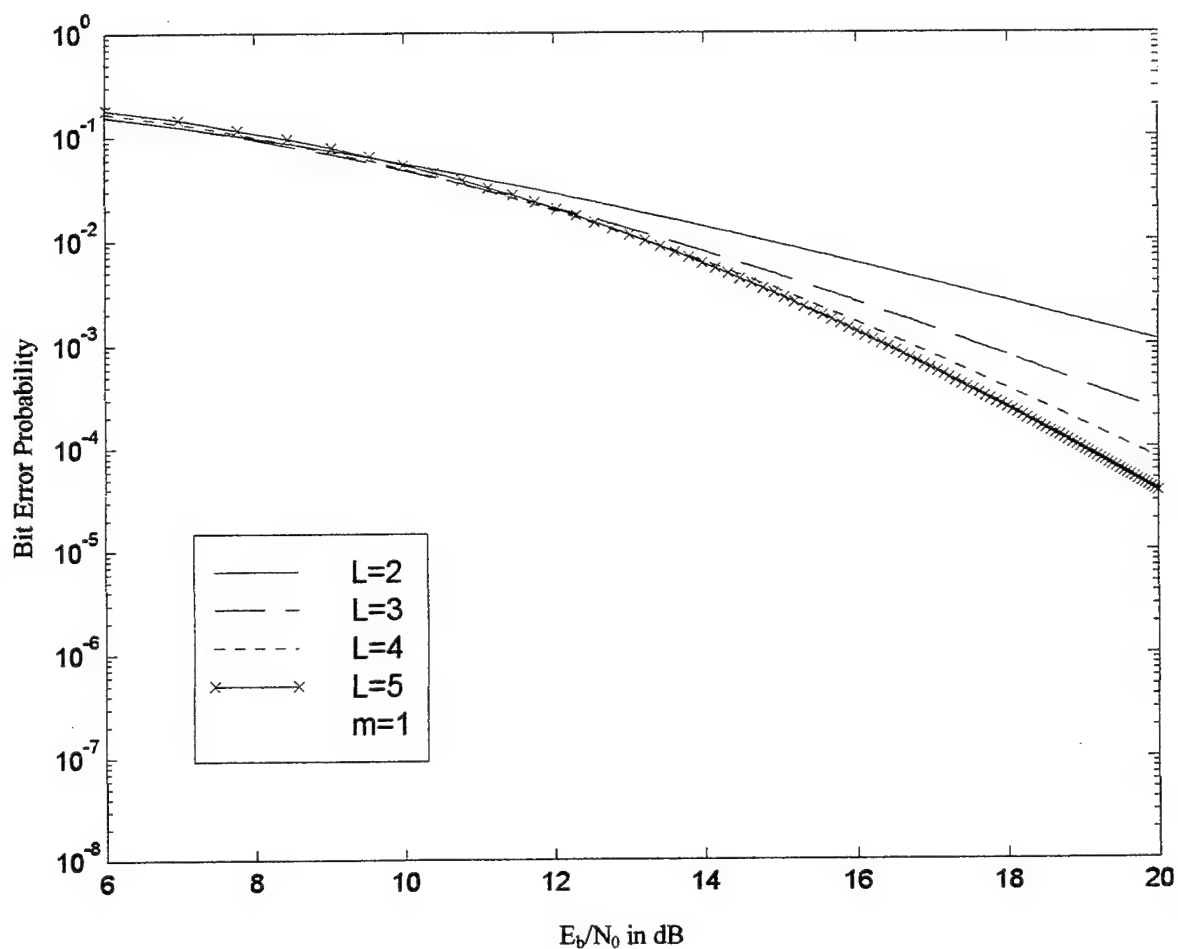


**Figure 11.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.5$ , using second order Selection Combining (SC-2) for diversity orders of  $L = 2, 3, 4$  and  $5$ .

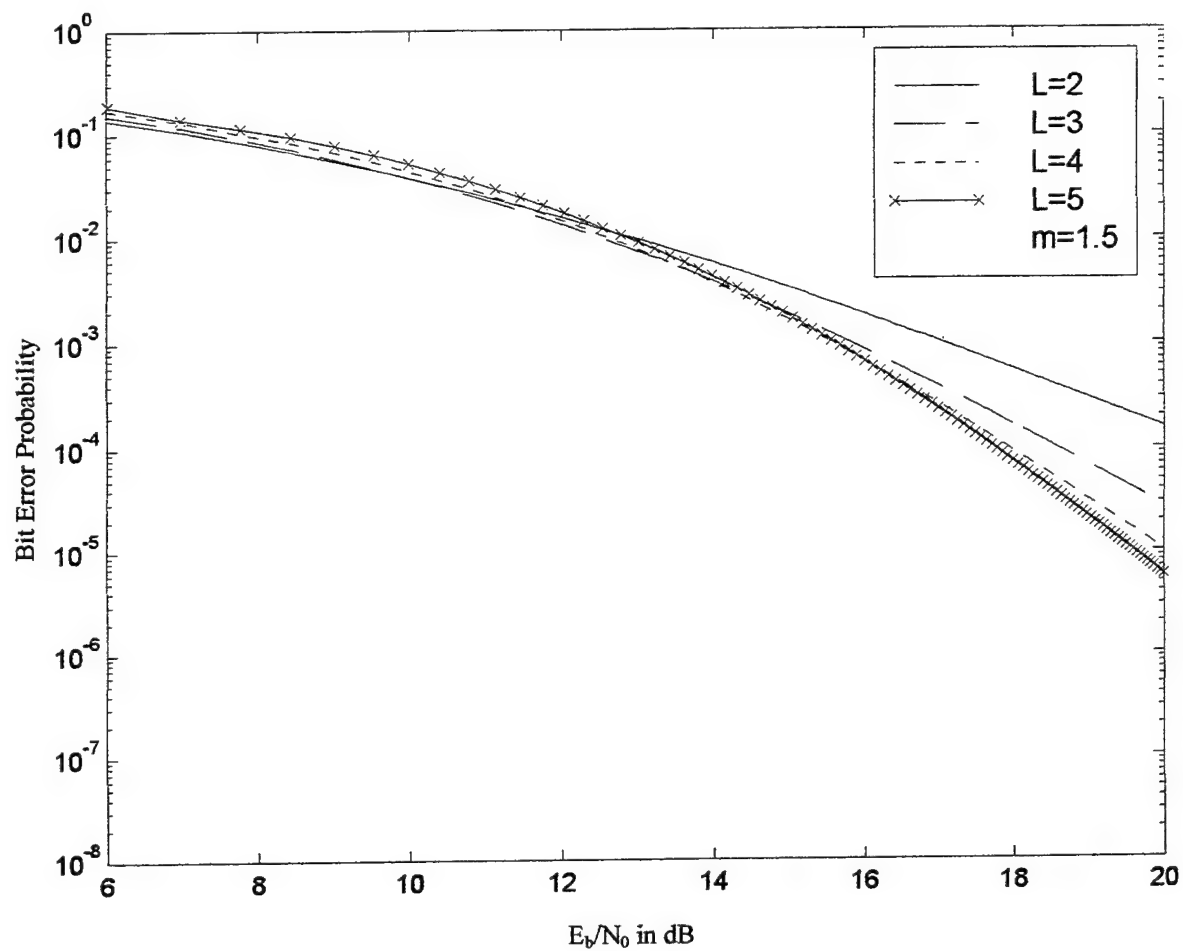




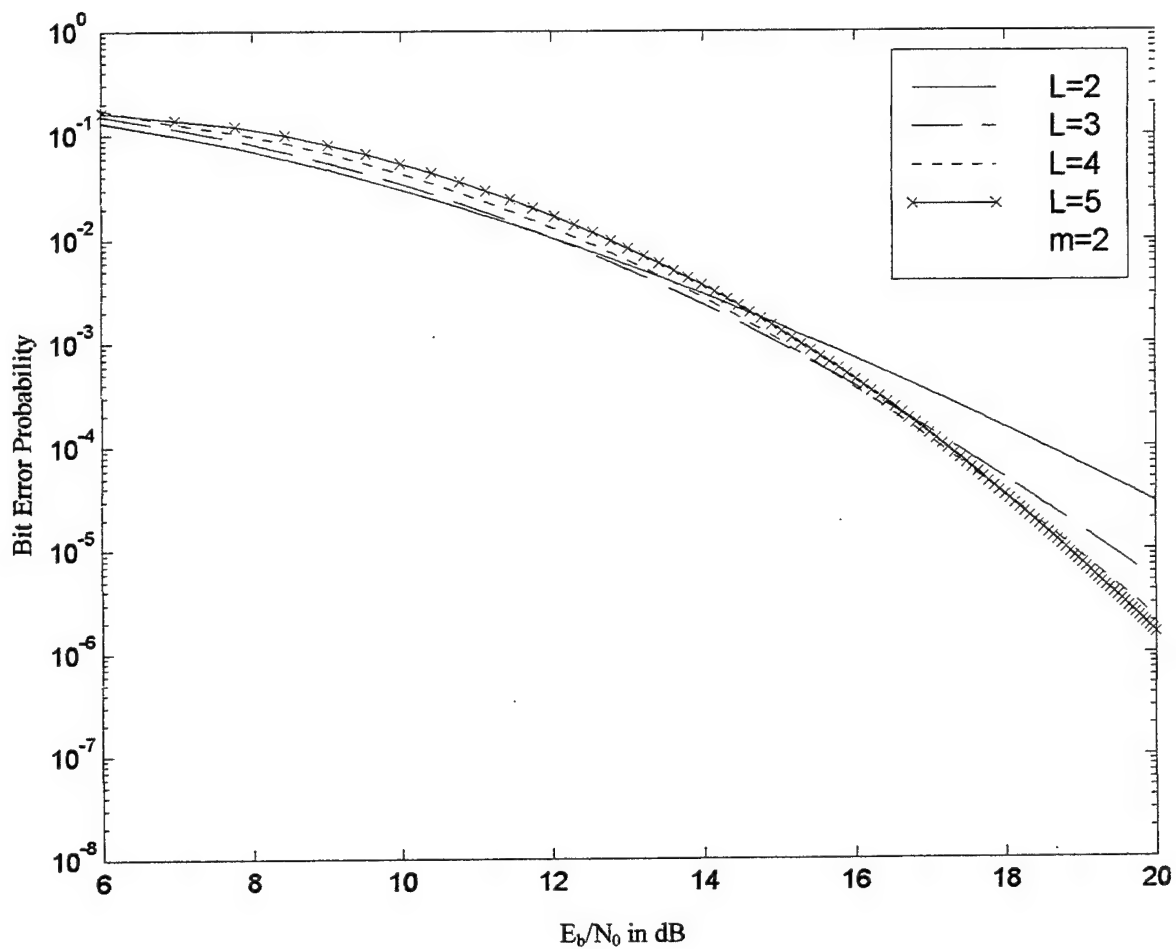
**Figure 12.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.75$ , using second order Selection Combining (SC-2) for diversity orders of  $L = 2, 3, 4$  and  $5$ .



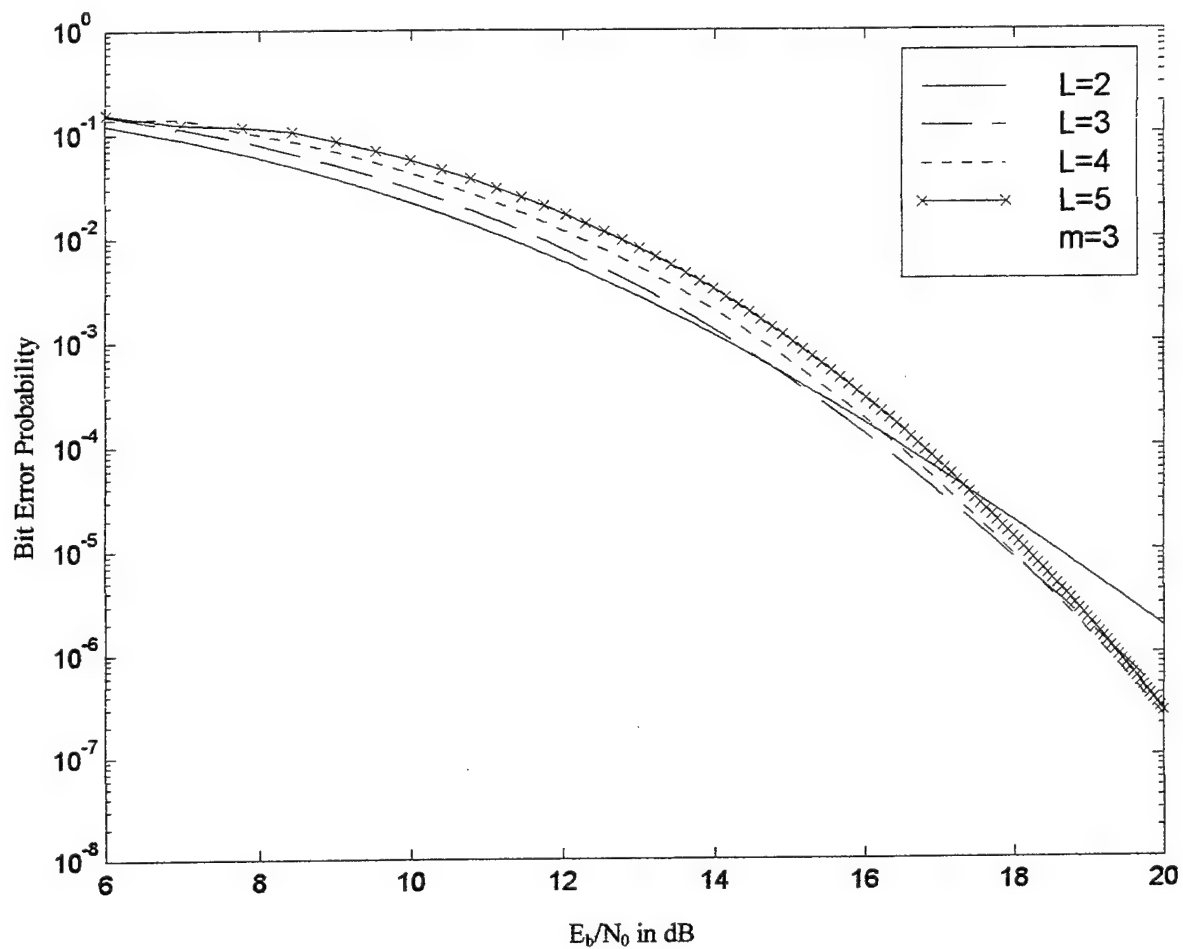
**Figure 13.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 1$ , using second order Selection Combining (SC-2) for diversity orders of  $L = 2, 3, 4$  and  $5$ .



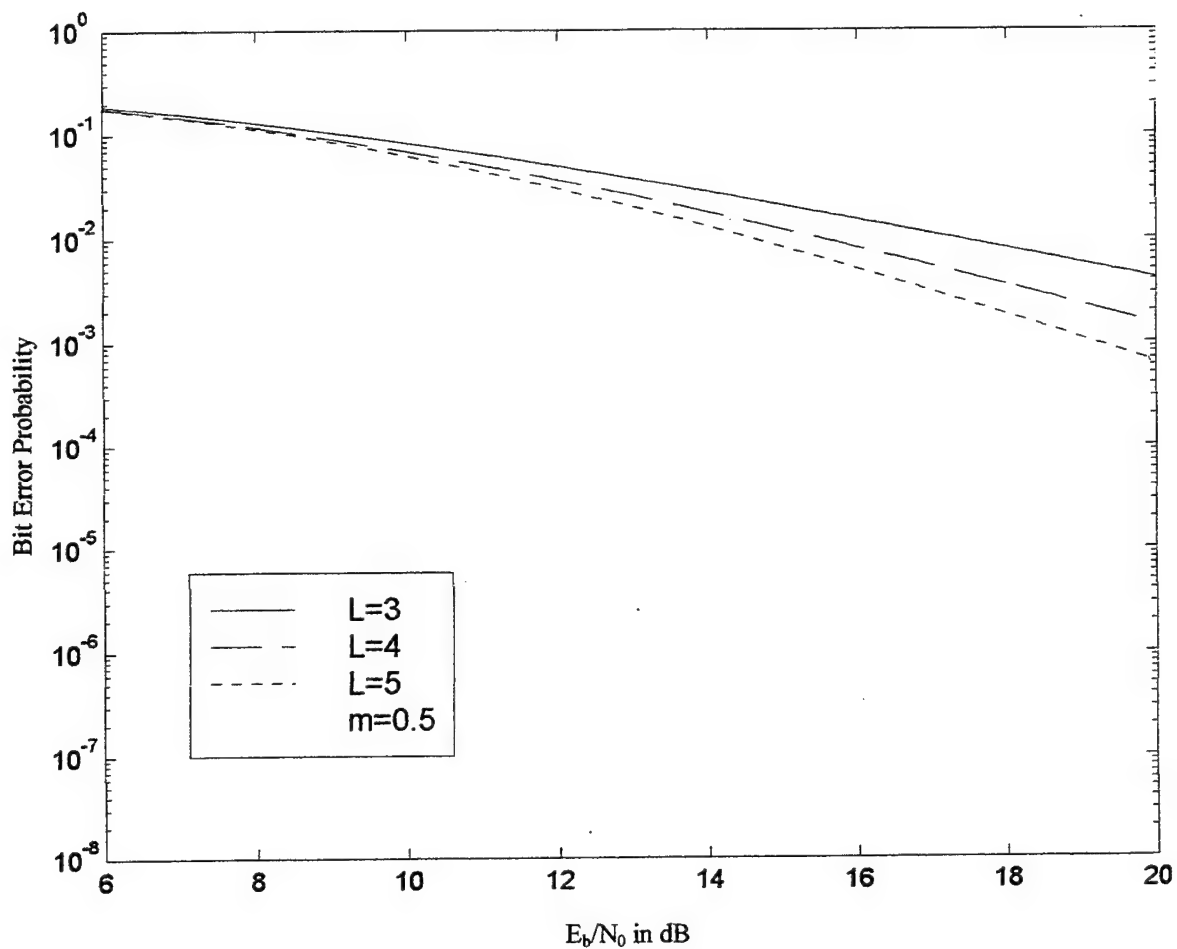
**Figure 14.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 1.5$ , using second order Selection Combining (SC-2) for diversity orders of  $L = 2, 3, 4$  and  $5$ .



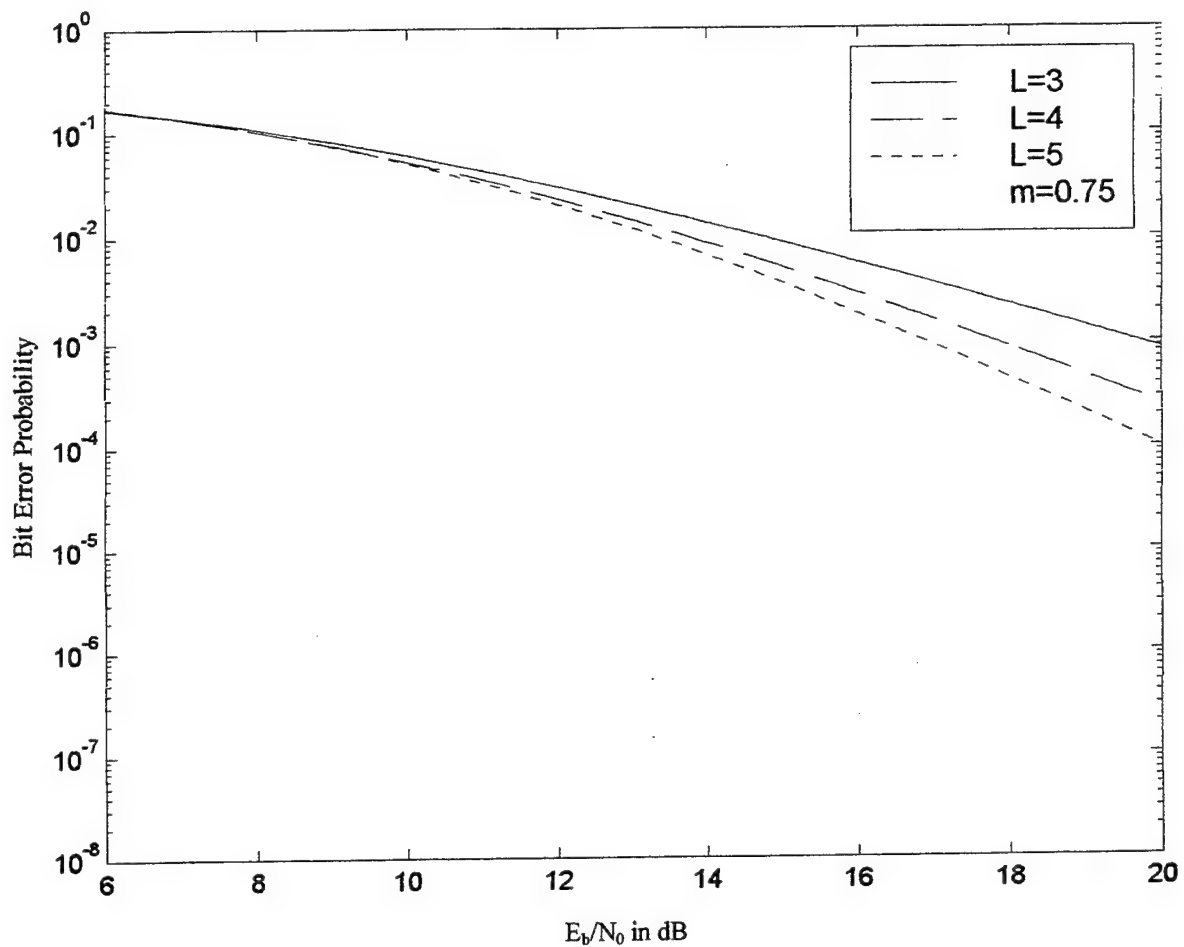
**Figure 15.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 2$ , using second order Selection Combining (SC-2) for diversity orders of  $L = 2, 3, 4$  and  $5$ .



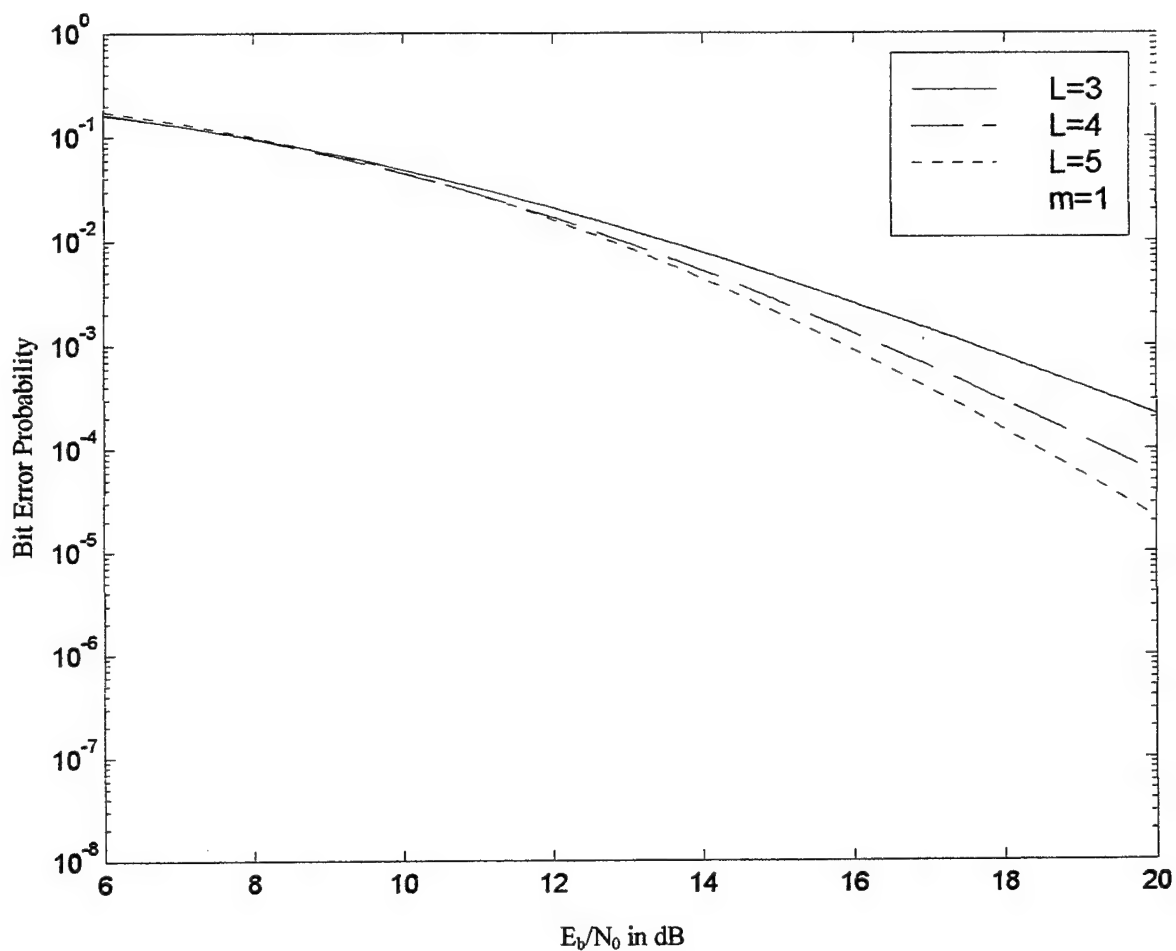
**Figure 16.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 3$ , using second order Selection Combining (SC-2) for diversity orders of  $L = 2, 3, 4$  and  $5$ .



**Figure 17.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.5$ , using third order Selection Combining (SC-3) for diversity orders of  $L = 3, 4$  and  $5$ .

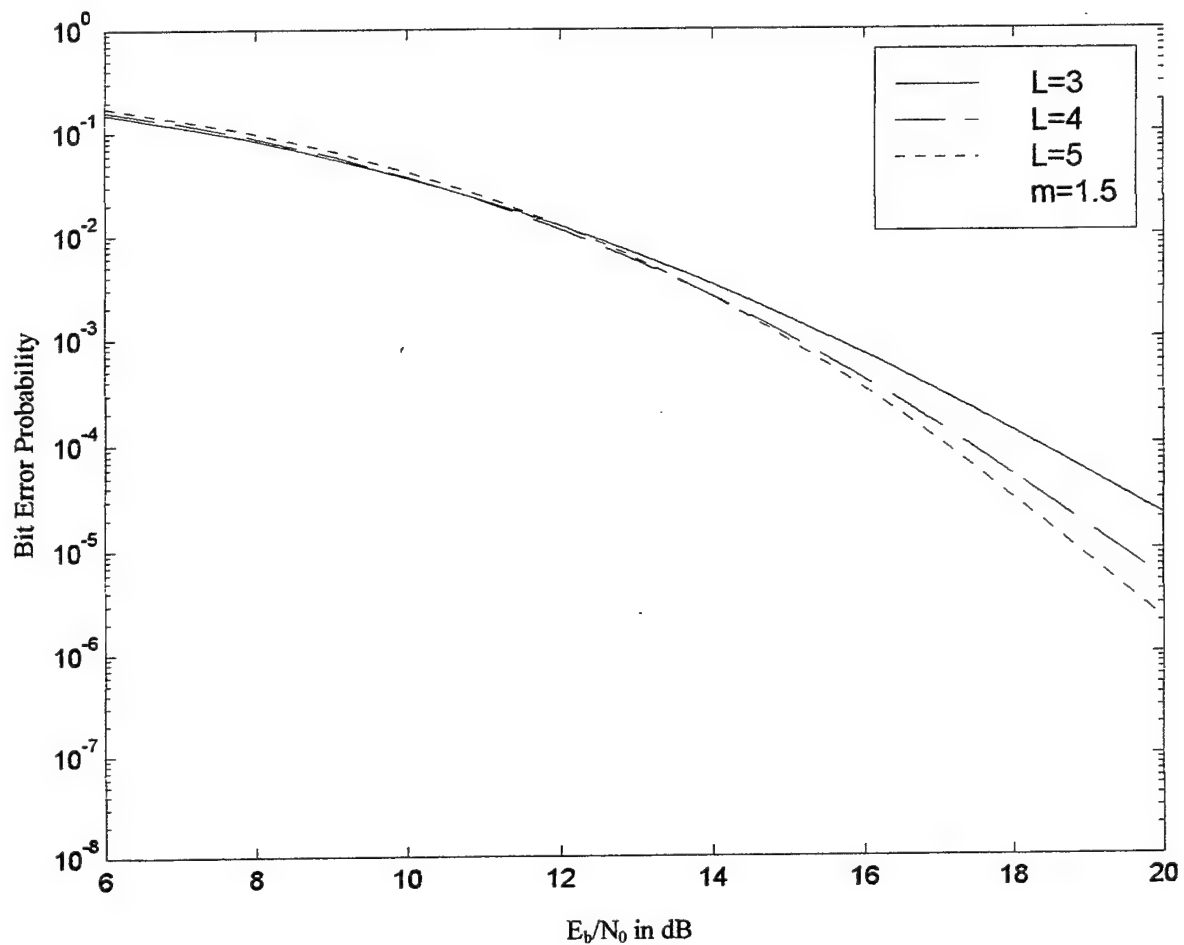


**Figure 18.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.75$ , using third order Selection Combining (SC-3) for diversity orders of  $L = 3, 4$  and  $5$ .

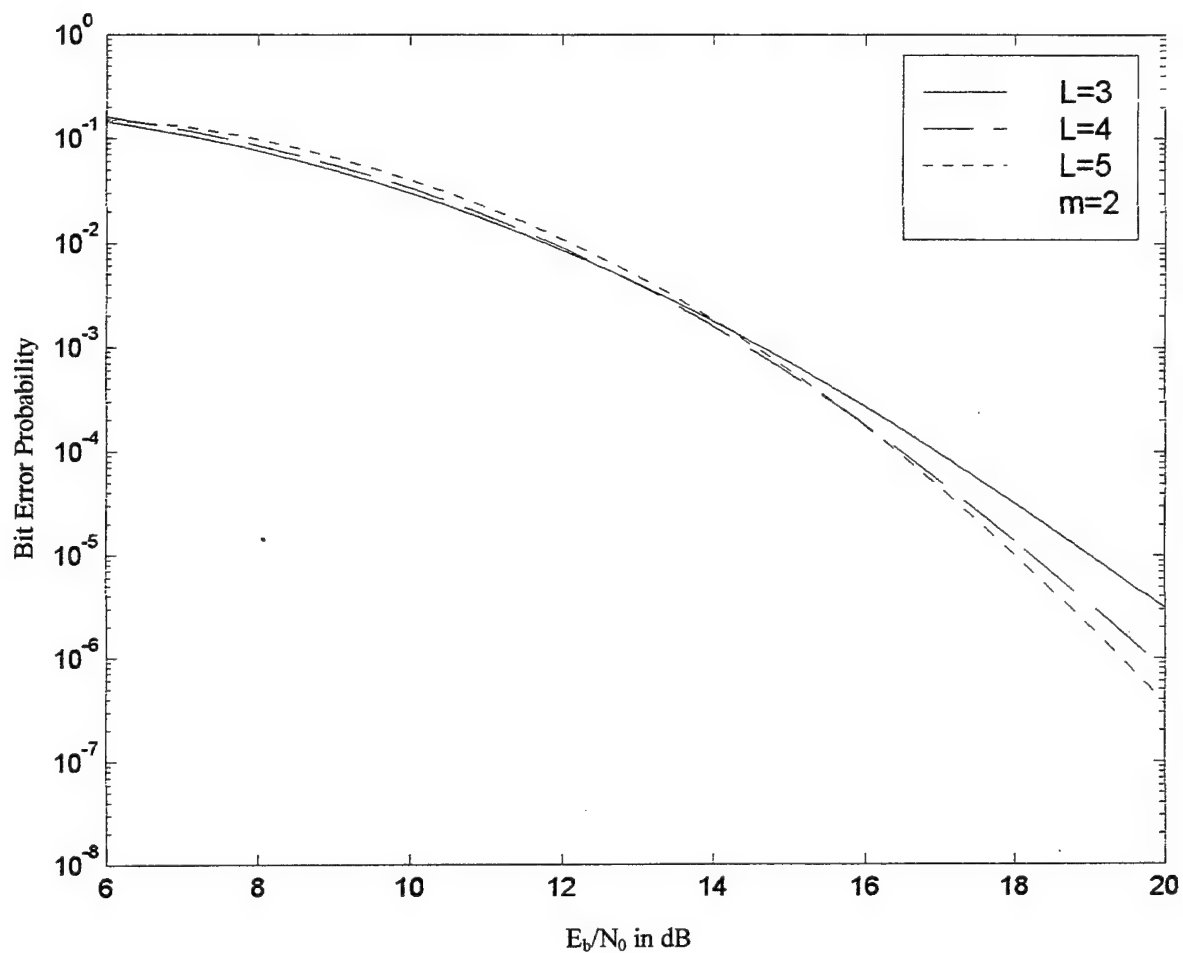


**Figure 19.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 1$ , using third order Selection Combining (SC-3) for diversity orders of  $L = 3, 4$  and  $5$ .

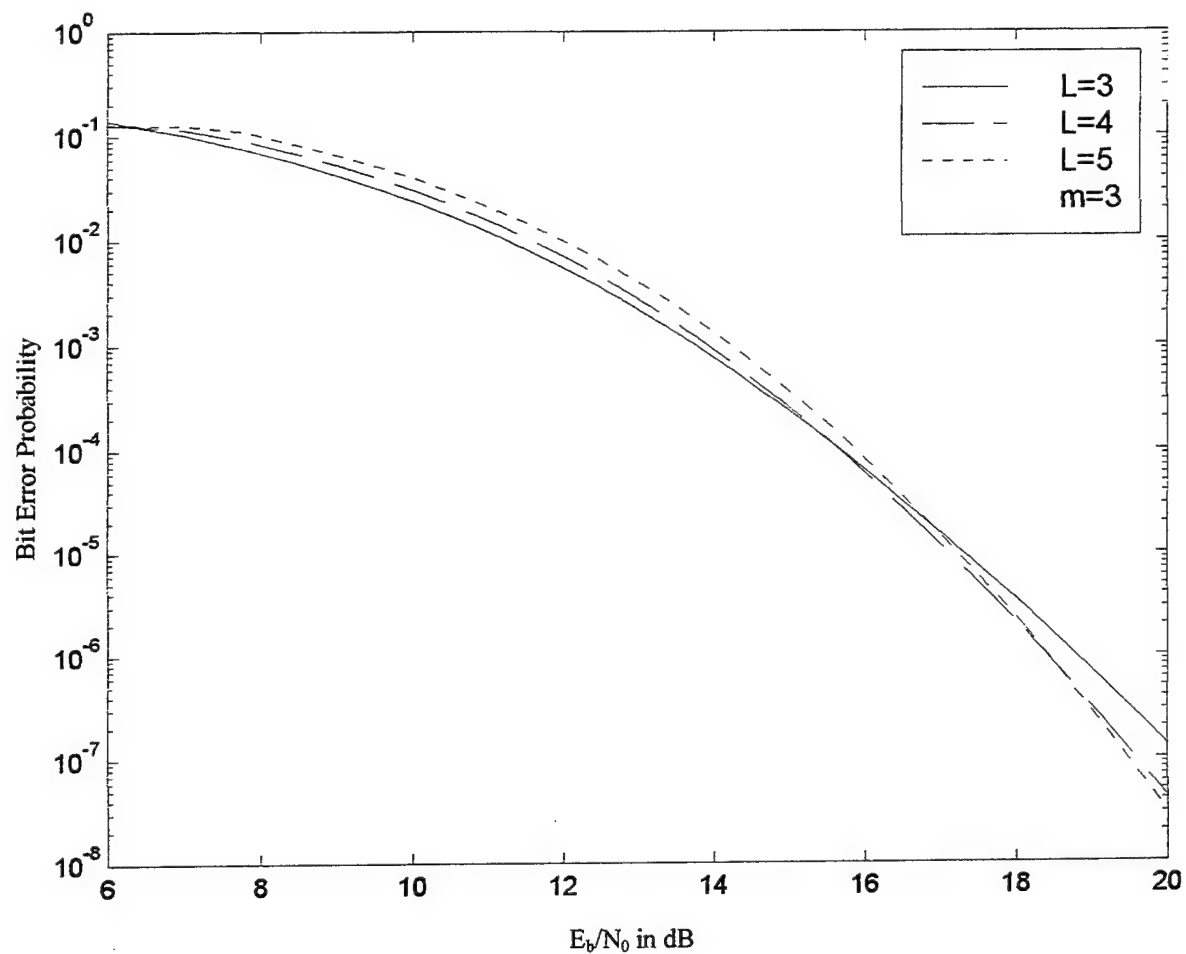




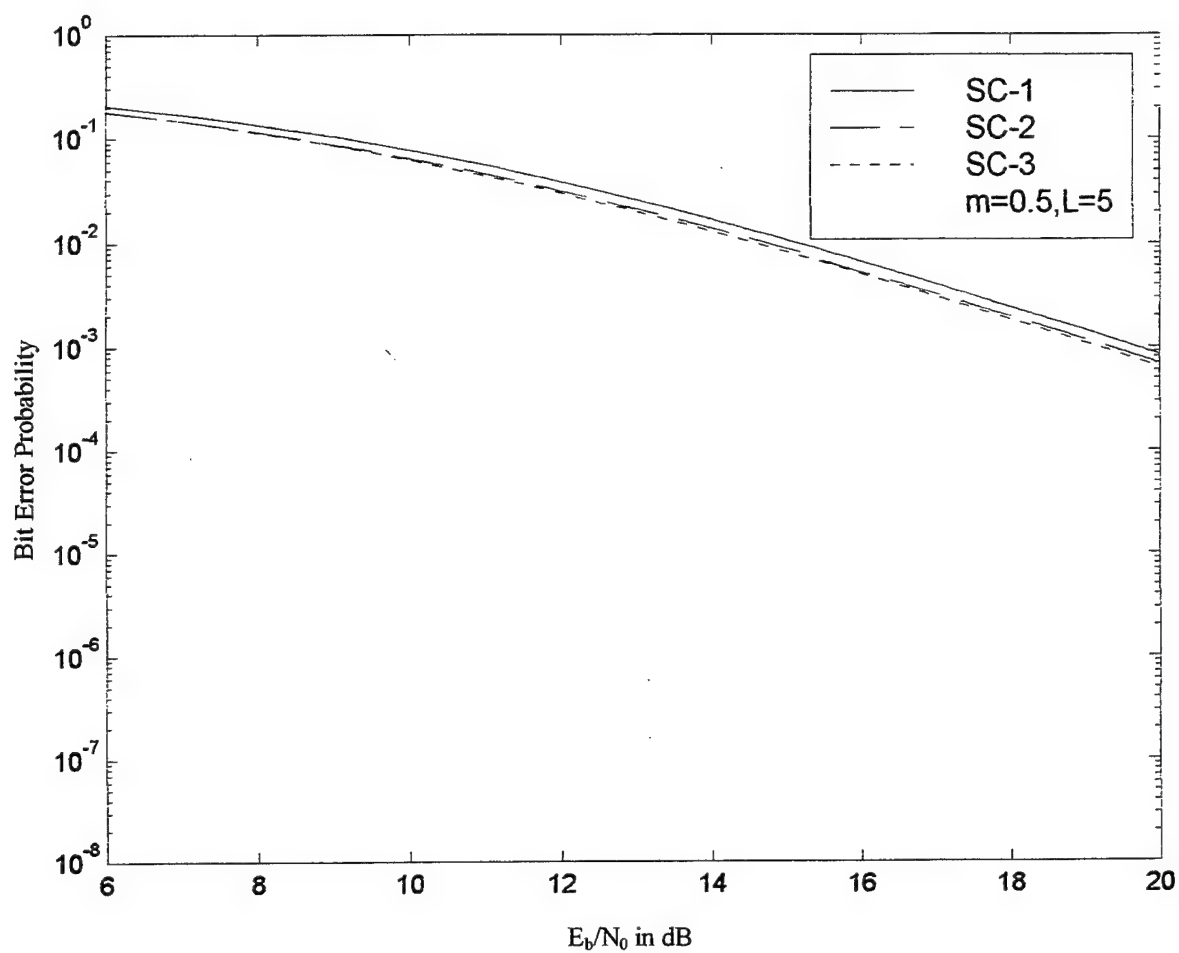
**Figure 20.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 1.5$ , using third order Selection Combining (SC-3) for diversity orders of  $L = 3, 4$  and  $5$ .



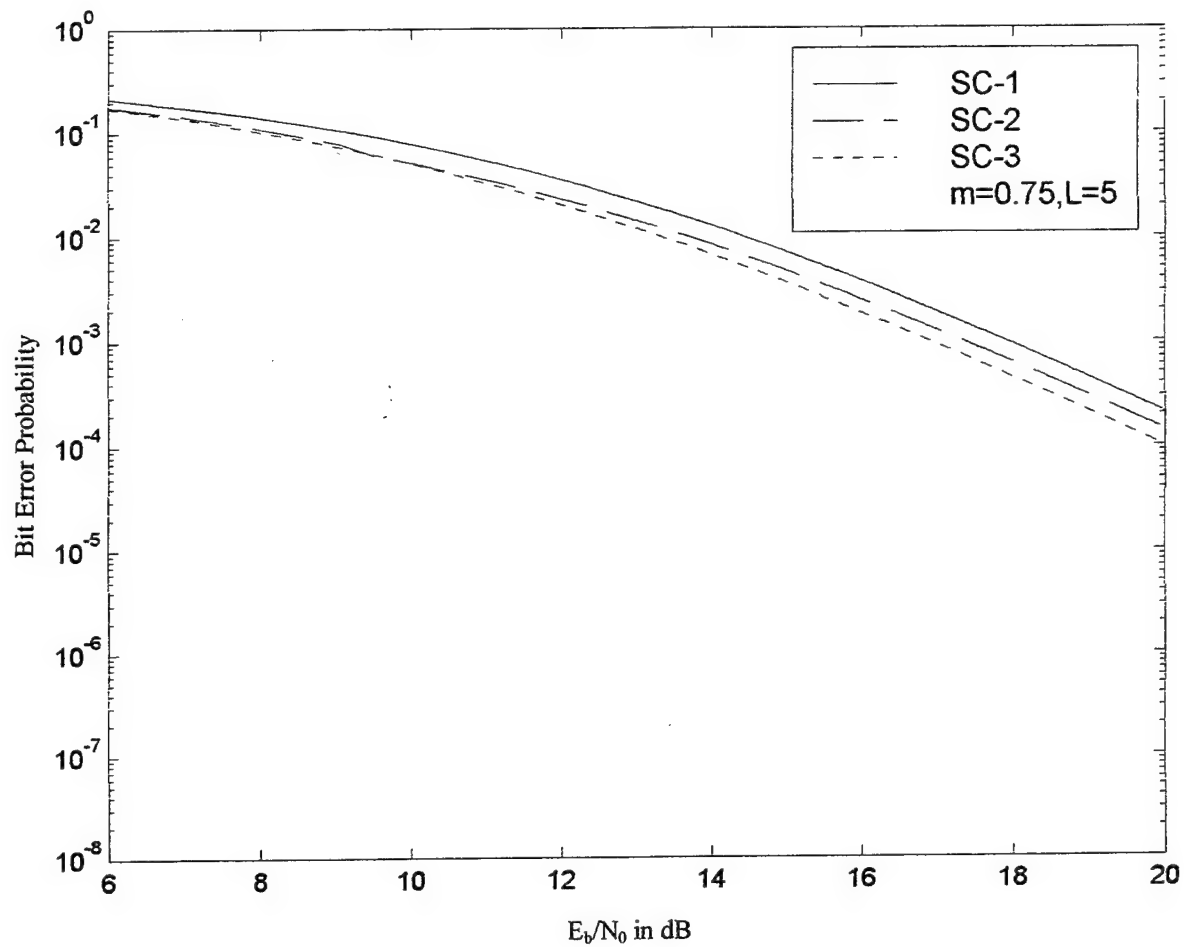
**Figure 21.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 2$ , using third order Selection Combining (SC-3) for diversity orders of  $L = 3, 4$  and  $5$ .



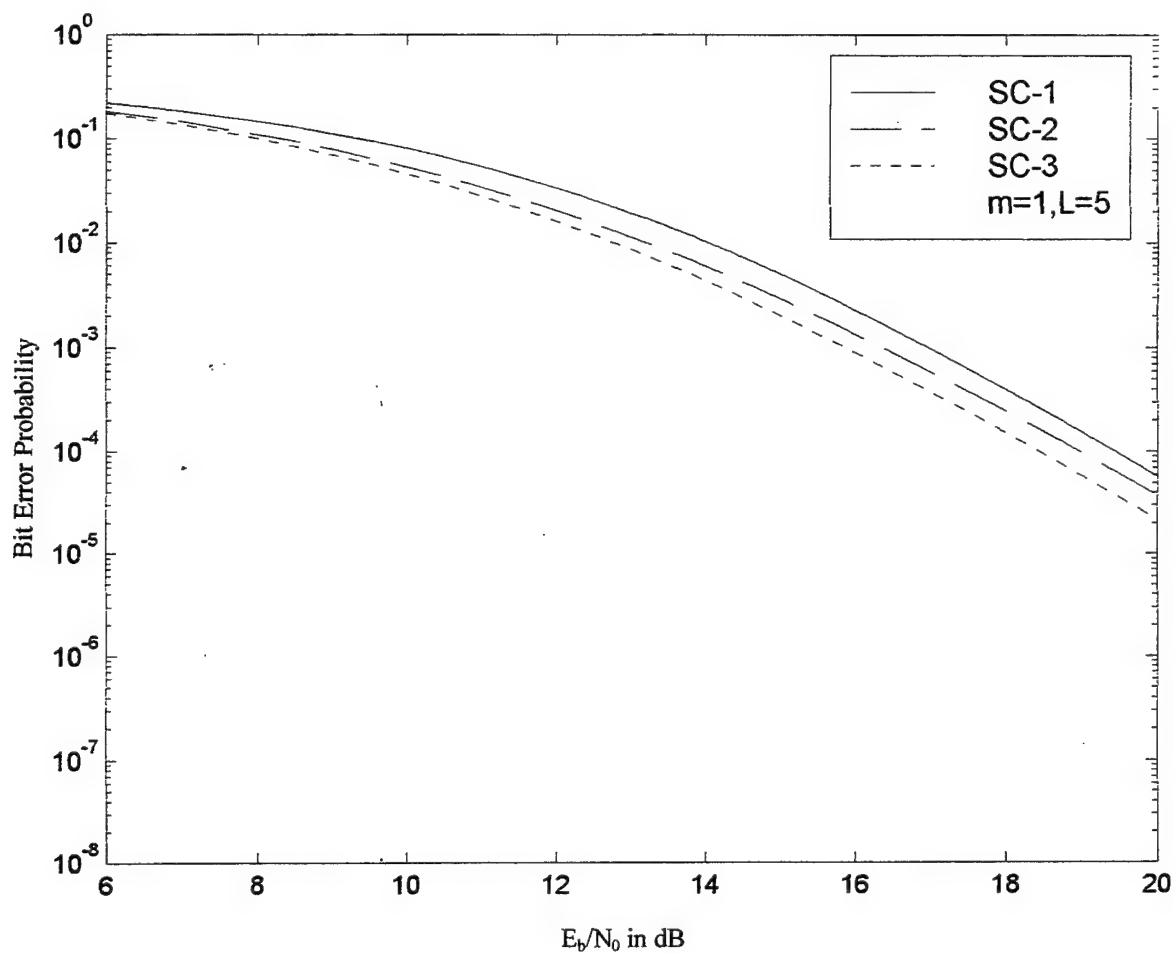
**Figure 22.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 3$ , using third order Selection Combining (SC-3) for diversity orders of  $L = 3, 4$  and  $5$ .



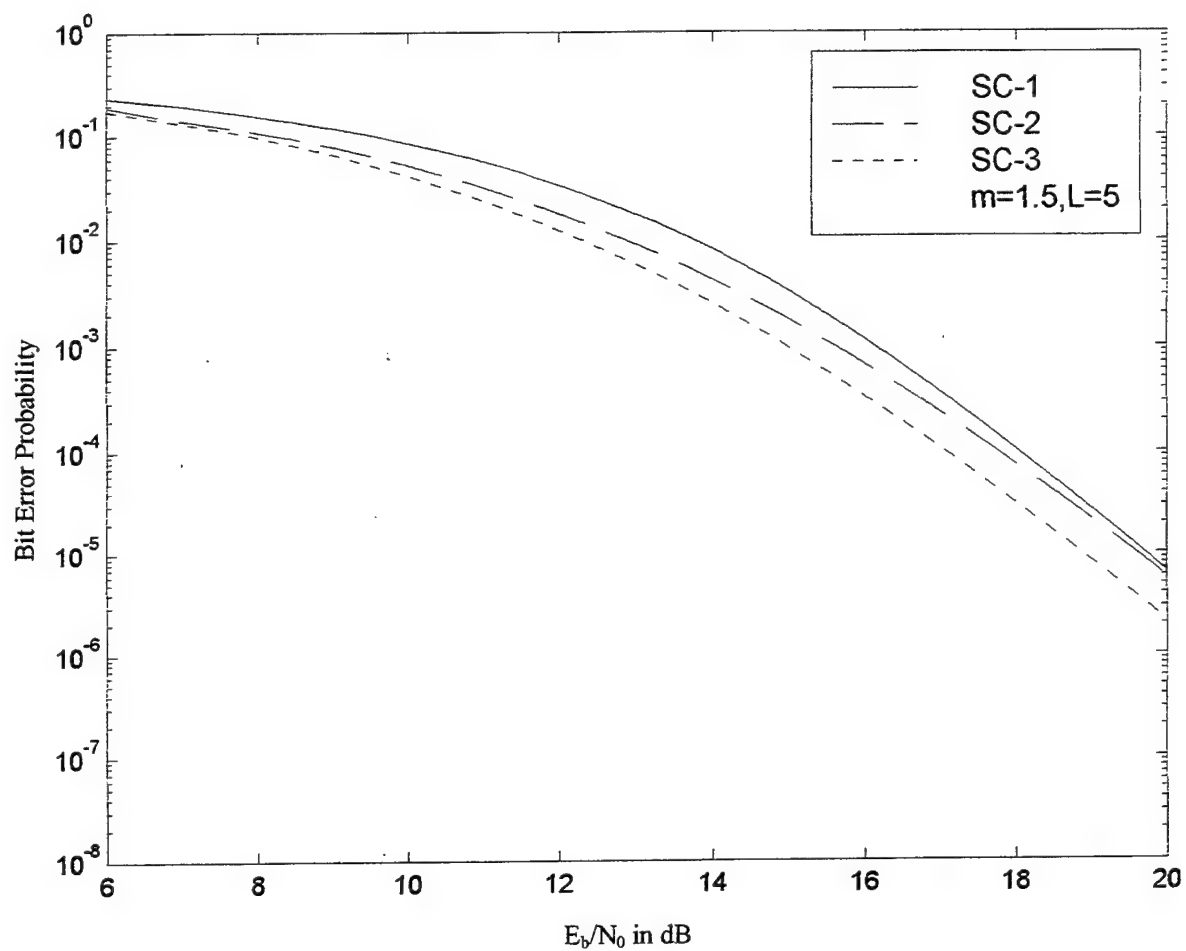
**Figure 23.** Receiver performance of SC, SC-2 and SC-3 over a Nakagami fading channel with  $m = 0.5$ , for diversity order of  $L = 5$ .



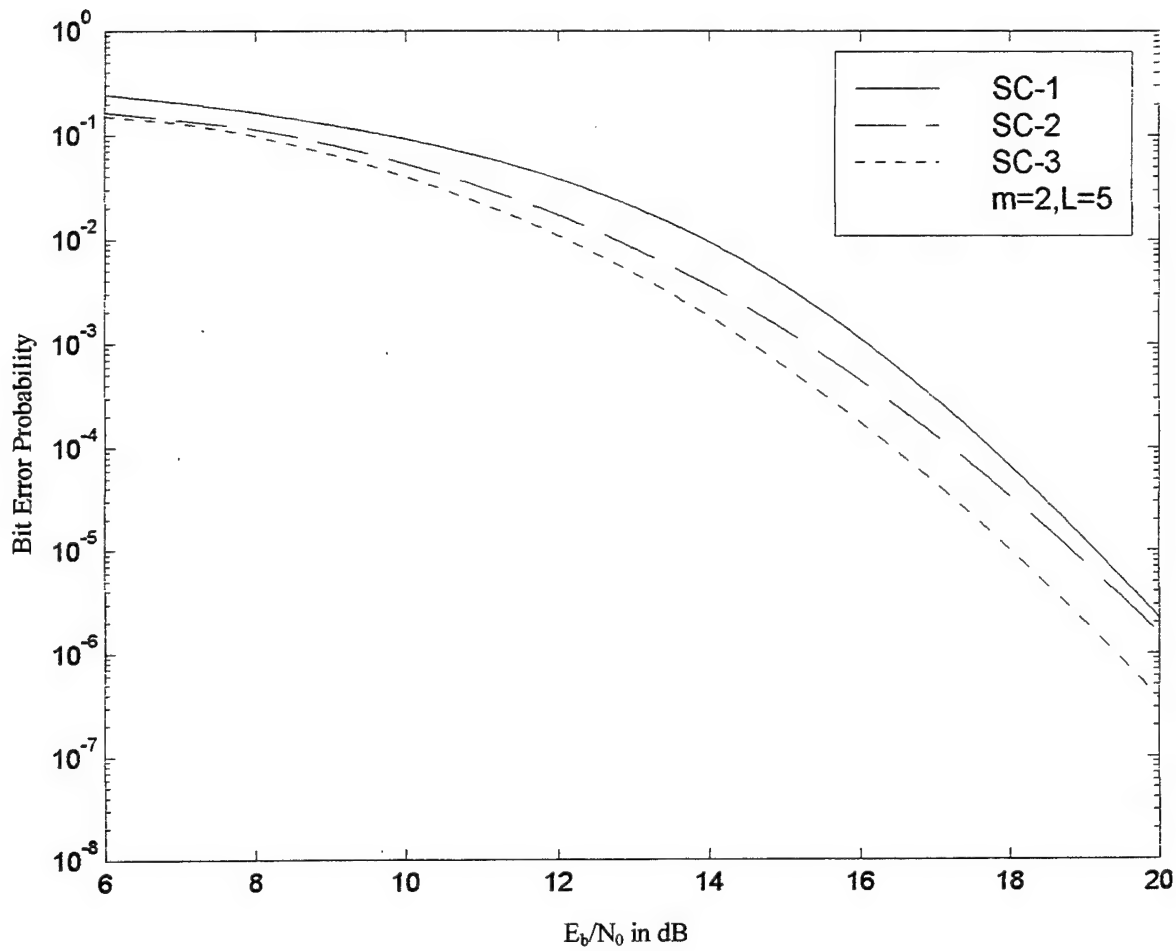
**Figure 24.** Receiver performance of SC, SC-2 and SC-3 over a Nakagami fading channel with  $m = 0.75$ , for diversity order of  $L = 5$ .



**Figure 25.** Receiver performance of SC, SC-2 and SC-3 over a Nakagami fading channel with  $m = 1$ , for diversity order of  $L = 5$ .

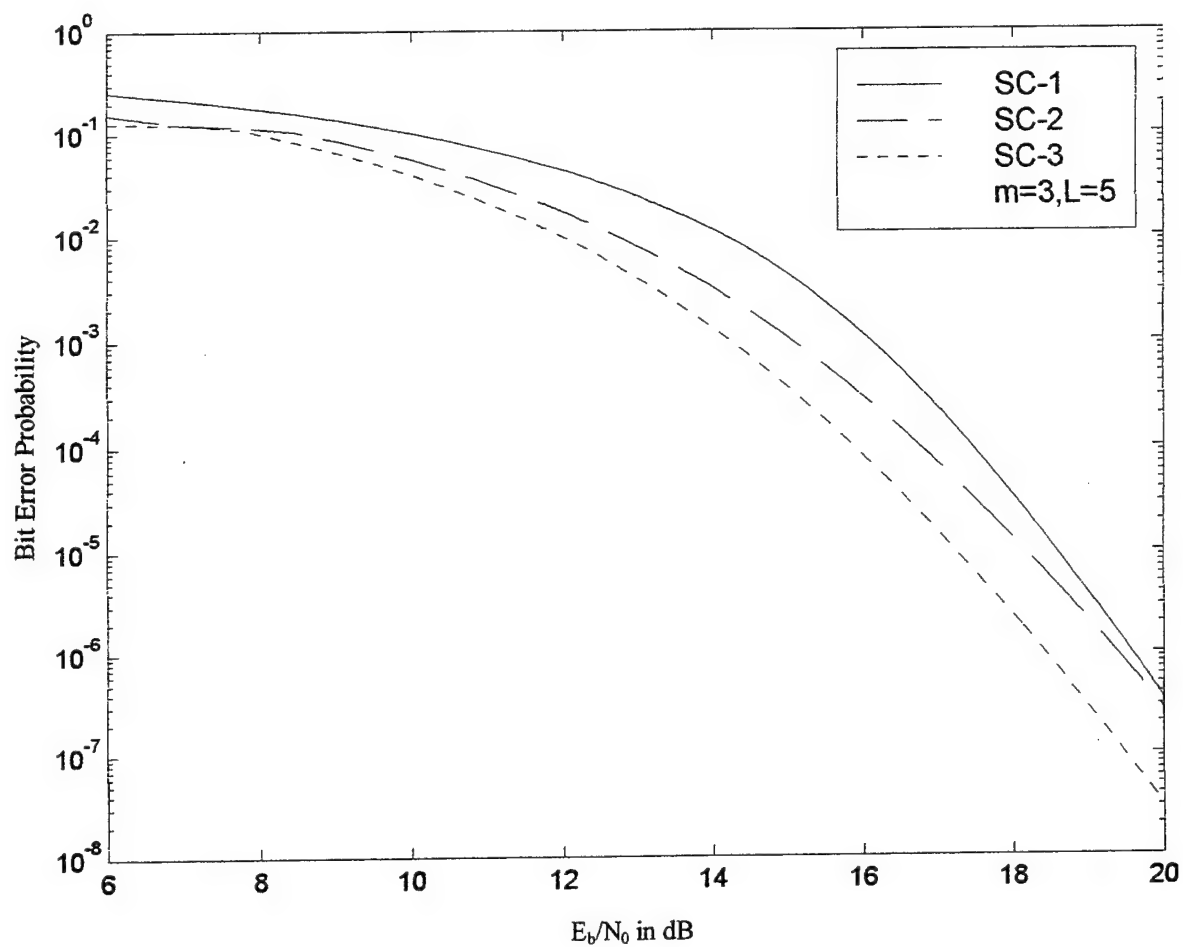


**Figure 26.** Receiver performance of SC, SC-2 and SC-3 over a Nakagami fading channel with  $m = 1.5$ , for diversity order of  $L = 5$ .

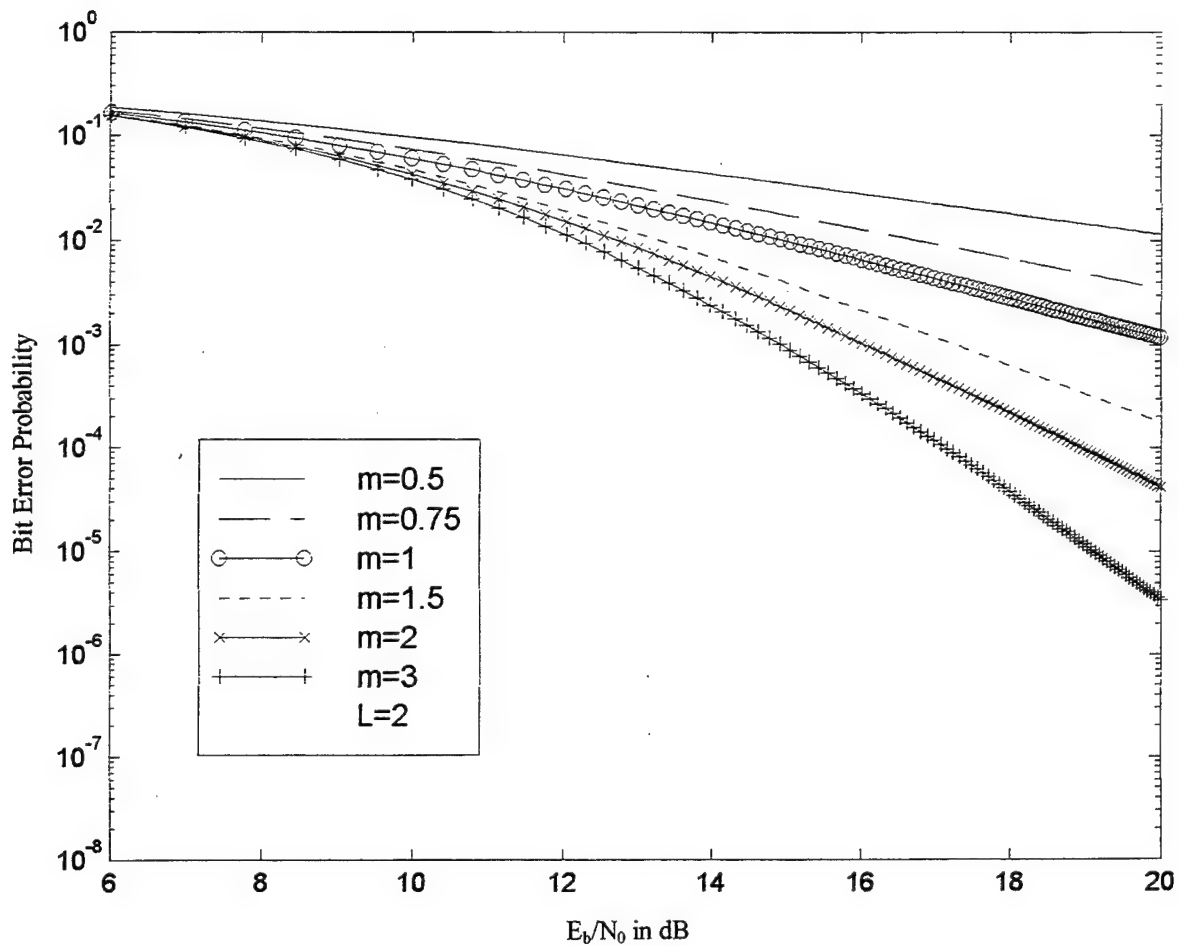


**Figure 27.** Receiver performance of SC, SC-2 and SC-3 over a Nakagami fading channel with  $m = 2$ , for diversity order of  $L = 5$ .

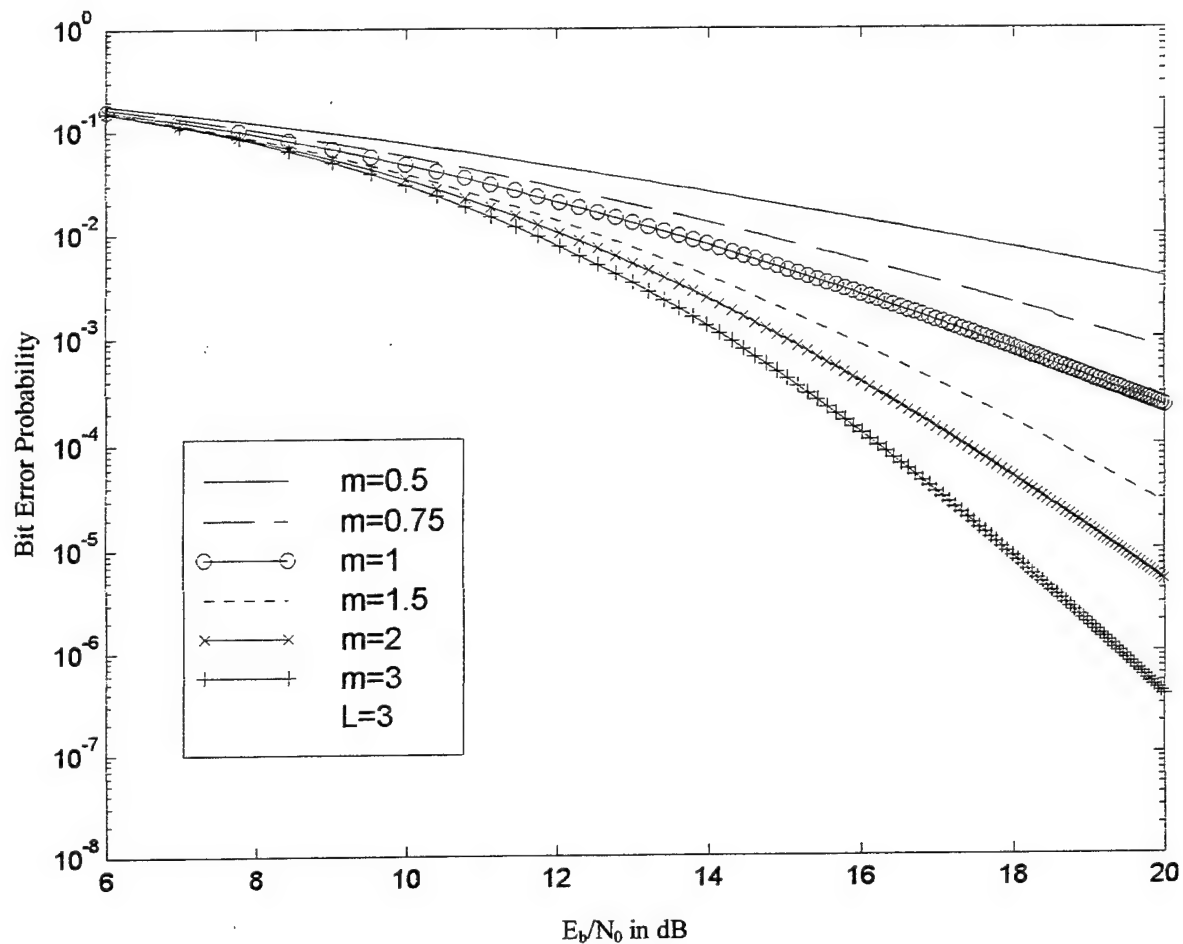




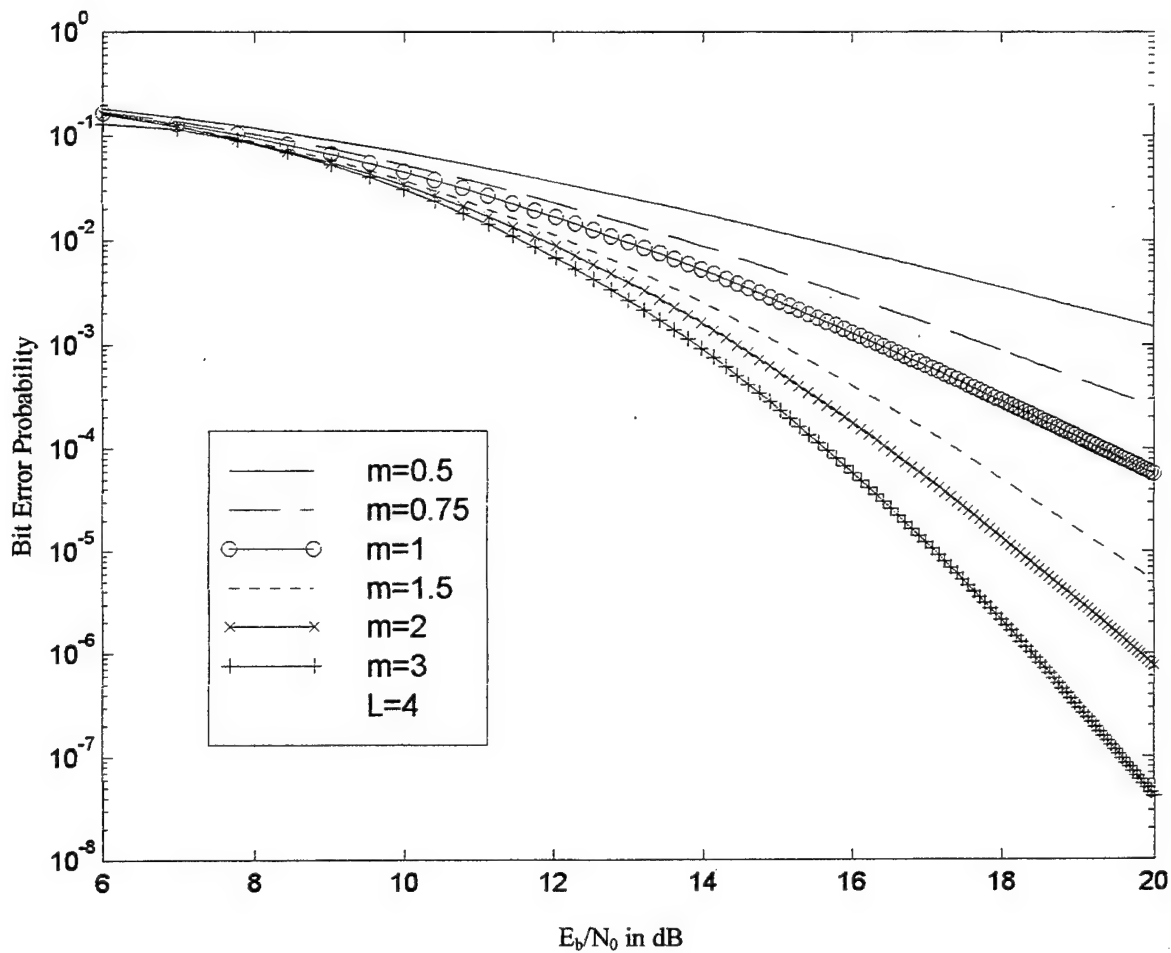
**Figure 28.** Receiver performance of SC, SC-2 and SC-3 over a Nakagami fading channel with  $m = 3$ , for diversity order of  $L = 5$ .



**Figure 29.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.5, 0.75, 1, 1.5, 2$  and  $3$  using first order Selection Combining (SC) for diversity order of  $L = 2$ .



**Figure 30.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.5, 0.75, 1, 1.5, 2$  and  $3$  using second order Selection Combining (SC-2) for diversity order of  $L = 3$ .



**Figure 31.** Performance of the non-coherent BFSK receiver over a Nakagami fading channel with  $m = 0.5, 0.75, 1, 1.5, 2$  and  $3$  using third order Selection Combining (SC-3) for diversity order of  $L = 4$ .



## VI. CONCLUSIONS

The objective of this thesis is the analysis of the Selection Combining techniques for a Binary FSK receivers operating over a frequency non-selective, slowly fading Nakagami channel. Numerical results are obtained to allow comparisons.

The SC techniques are in general simple techniques because they can give satisfactory performance without an  $L$  dependency. This is something we desire in order to construct simpler receivers. On the other hand these techniques are not optimal techniques, since they do not use all the available diversity branches at the same time. But if the diversity order  $L$  varies as a function of location and time, it is desirable that the receiver have an  $L$  independency.

It is shown that a system with a higher diversity order  $L$  performs better than a system with a smaller  $L$ . As far as the comparison between the three techniques is concerned, we conclude that as the order increases the Selection Combining techniques perform better. Finally regarding the Nakagami fading channel, it is shown that as the factor  $m$  increases the system performs better .



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